IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received August 2, 2019, accepted August 31, 2019, date of publication September 9, 2019, date of current version September 24, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2940104

A Novel Candidate Solution Generation Strategy for Fruit Fly Optimizer

HAZIM ISCAN, MUSTAFA SERVET KIRAN, AND MESUT GUNDUZ
Department of Computer Engineering, Faculty of Engineering, Selcuk University, 42075 Konya, Turkey
Department of Computer Engineering, Faculty of Engineering and Natural Sciences, Konya Technical University, 42075 Konya, Turkey

Corresponding author: Hazim Iscan (hiscan@ktun.edu.tr)

This work was supported by the Selcuk University/Konya Technical University Scientific Project Coordinatorship under Grant 18101009.

ABSTRACT Fruit fly optimization algorithm (FOA) is one of the swarm intelligence algorithms proposed for solving continuous optimization problems. In the basic FOA, the best solution is always taken into consideration by the other artificial fruit flies when solving the problem. This behavior of FOA causes getting trap into local minima because the whole population become very similar to each other and the best solution in the population during the search. Moreover, the basic FOA searches the positive side of solution space of the optimization problem. In order to overcome these issues, this study presents two novel versions of FOA, pFOA_v1 and pFOA_v2 for short, that take into account not only the best solutions but also the worst solutions during the search. Therefore, the proposed approaches aim to improve the FOA's performance in solving continuous optimizations by removing these disadvantages. In order to investigate the performance of the novel proposed FOA versions, 21 well-known numeric benchmark functions are considered in the experiments. The obtained experimental results of pFOA versions have been compared with the basic FOA, SFOA which is an improved version of basic FOA, SPSO2011 which is one of the latest versions of particle swarm optimization, firefly algorithm called FA, tree seed algorithm TSA for short, cuckoo search algorithm briefly CS, and a new optimization algorithm JAYA. The experimental results and comparisons show that the proposed versions of FOA are better than the basic FOA and SFOA, and produce comparable and competitive results for the continuous optimization problems.

INDEX TERMS Fruit fly algorithm, best-worst strategy, continuous optimization, numeric benchmark problem.

I. INTRODUCTION

In recent years, the behaviors of the biological creatures found in the nature are inspiration of the researchers for development novel optimization algorithms and many novel optimization algorithms have been developed by considering these behaviors. These algorithms are called metaheuristic algorithms. Metaheuristic algorithms are mostly based on swarm intelligence or evolutionary computation and they are used to solve optimization problems with an alternative of classical optimization techniques. Due the structure of the optimization problems, the optimization can be a time consuming and complicated process, and classical techniques are generally problem-dependent. Metaheuristic algorithms can solve optimization problems with different characteristics and they need a little modification because they are general-purpose optimizers. Moreover, despite the fact that they cannot guarantee the optimal solution, they guarantee optimal or suboptimal solutions in a reasonable time and efforts. Some of the swarm intelligence algorithms are particle swarm optimization (PSO) [1], firefly algorithm (FA) [2], cuckoo algorithm (CS) [3], ant colony algorithm (ACO) [4], artificial bee colony algorithm (ABC) [5], fruit fly optimization algorithm (FOA) [6], tree seed algorithm (TSA) [7].

The FOA is one of the recently proposed swarm intelligence algorithm and it models the behavior of food search behaviors of fruit flies, which was developed by Pan in 2012 [6]. The important features of FOA are that it has a simple algorithmic structure, few parameters for adjusting, ease of understand and application and fast convergence characteristics. Based on these advantages, it has been applied to solve many optimization problems. For example, financial distress [6], [8]–[10], power load forecasting [11]–[13], scheduling problem [14]–[19], key control

The associate editor coordinating the review of this manuscript and approving it for publication was Sotirios Goudos.

characteristic optimization [20], PID controller [21]–[25], semiconductor final testing scheduling [26], electricity consumption forecasting [27], web auction logistics service [28], surface vehicles application [29], optimization of LSSVM parameters [30], [31], GRNN optimization [32]–[35], in order to eliminate noise components from machinery sound [36], [37], optimal placement of phasor measurement units [38], knapsack problem [39], [40], joint replenishment problems [41], clustering parameter problems [42], range image registration [43], analysis of large antenna array [44], [45], acoustic-based cutting pattern recognition [46], wind speed forecasting [47]. In order to improve performance of FOA, researchers proposed its variants by modifying structure of the algorithm. Pan [48] proposed the MFOA, which uses a three-dimensional search field. Shan *et al.*[49] presented the LGMS-FOA, which uses the linear production mechanism. Pan *et al.* [50] proposed an improved version of IFFO with adaptive control parameter. Yuan *et al.* [51] used multiple swarm behaviors in FOA. Zhang *et al.* [52] presented MSFOA, which avoided local optima using the gauss mutation operator. Cong *et al.* [53] used the least square support vector machine LSSVM and FOA together to solve the traffic flow estimation problem. Lin [28] presented an approach that uses the FOA together with the general regression neural network. Wang *et al.* [39] suggested a binary FOA to solve the knapsack problem. Zheng and Wang [54] proposed a two-stage adaptive fruit fly optimization algorithm (TAFOA) to solve unrelated parallel machine scheduling problem. Lv *et al.* [55] improved their swarm diversity and search capability with their hybrid location mechanism. Xiao *et al.* [56] presents an improved FOA based on the cell communication mechanism (CFOA). Wang et al. proposed JS-FOA, developed through common search strategies driven by biological memory [57].

Despite these advantages, applications and modification of the basic FOA, it has some important disadvantages such as positive search, stagnation and premature convergence. Due to mathematical model of the algorithm, the search is done on the positive side of solution space. Because whole population follows the best solution in the population, the artificial agents become very similar to each other during the optimization. This is useful for convergence to the acceptable solutions, however these cause a stagnation in the population and sometimes premature convergence, especially multimodal optimization problems. When the algorithms is analyzed individually, it is seen that the FOA algorithm is successful in solving the unimodal optimization problems due to fact that its solution generation mechanism is based on the following the best solution in the population. So FOA is good at unimodal optimization with speed convergence characteristics. The JAYA algorithm uses best and worst solution in the population while a candidate solution is generated. This JAYA's mechanism is useful in discovering potential optimal point in the solution space. Therefore, it is good at solving multimodal optimization problems but its convergence characteristics in the unimodal problem questionable. By combining the FOA with JAYA, we try to obtain better algorithm in solving both unimodal and multimodal optimization problems with acceptable convergence speed. This is the innovation and approach background of the manuscript. The peculiar property of JAYA is the usage of the worst solution in the solution generation mechanism. While more algorithms use the best solution in order to improve local search capability and convergence characteristics, the JAYA use best and worst solution in the solution generation mechanism in order to improve local and global search capabilities. In order to improve global search capability of the FOA, we integrated the solution generation of JAYA with the FOA. In this study, we propose a best-worst-based solution update strategy in the JAYA by modifying its algorithmic structure in order to improve the performance of the algorithm on continuous optimization.

The paper is organized as follows: the study is introduced in Section 1 and a brief literature review is also given in this section. In Section 2 and 3, we give the explanations on basic FOA and JAYA Algoritm and the proposed methodology is presented in Section 4. The experiments are designed in Section 5 and obtained results are discussed in Section 6. The study is concluded in Section 7 and at the same time a future directions is presented in this section.

II. BASIC FRUIT FLY OPTIMIZATION ALGORITHM

The basic version of FOA has been proposed for solving nonlinear global optimization by inspiring the food search behavior of real fruit flies. Fruit flies live in tropical regions, and their smell and vision organs is more capable than the other fly species. Furthermore, it can get the smell of the food that is 40 km away. Firstly, it perceives the smell of food and flies to that area. Then it finds food with a sensitive vision [6]. The FOA algorithm has few parameters with respect to other swarm intelligence algorithms and the algorithmic structure is simple, ease to understand and program. The basic algorithm attracts more attentions because of these features and it is used to solve many optimization problems. The search behavior of fruit flies in the algorithm is given in Fig.1.

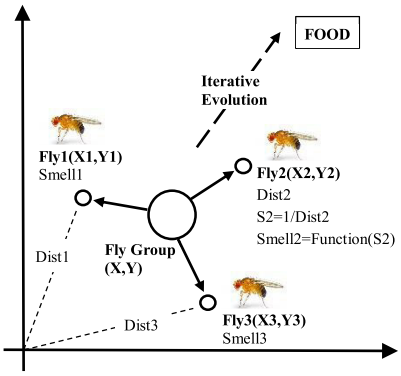

FIGURE 1. The search behavior of fruit flies.

FOA is composed of two phases as smell and vision. In the smell phase, it takes the smell of the food and flies

Algorithm. pFOA
Parameters. Population size (N), dimension of the problem (D), maximum iterations ($maxgen$) and boundaries of the problem($maxX$ and $maxY$)
Output. Solution X

// Initialize the initial swarm positions
for j=1,2,…, D
  $x_{0_j} = maxX * (2 * rand - 1)$
  $y_{0_j} = maxY * (2 * rand - 1)$
endfor

// Initialize the population and find the best solution
for i=1,2,…, N
  for j=1,2,…, D
    $x_{i,j} = x_{0_j} + (2 * rand - 1)$
    $y_{i,j} = y_{0_j} + (2 * rand - 1)$
    $d_{i,j} = \sqrt{x_{i,j}^2 + y_{i,j}^2}$
    $S_{i,j} = \frac{1}{d_{i,j}}$
  endfor
  $f_i = fitness(\overrightarrow{S_i})$
endfor

// Iterative evoluation
for *gen*=1,2,…, *maxgen*

  $[bestS\ bi] = min(f_i)$
  $\overrightarrow{Best} = \overrightarrow{S_{(bi)}}$
  $[worstS\ wi] = max(f_i)$
  $\overrightarrow{Worst} = \overrightarrow{S_{(wi)}}$

  if *gen is in every 20 percent of maxgen*
    $\overrightarrow{S_{axis}} = \overrightarrow{S_{best}}$
    for i=1,2,…, N
      $S_{i,j} = S_{axis,j} + rand_1 * (Best_j - |S_{axis,j}|) - rand_2 * (Worst_j - |S_{axis,j}|)$ *// for pFOA_v1*
      $S_{i,j} = S_{axis,j} + rand_1 * Best_j - rand_2 * Worst_j$ *// for pFOA_v2*
    endfor
  endif

  for i=1,2,…, N
    for j=1,2,…, D
      $Snew_{i,j} = S_{i,j} + rand_1 * (Best_j - |S_{i,j}|) - rand_2 * (Worst_j - |S_{i,j}|)$ *// for pFOA_v1*
      $Snew_{i,j} = S_{i,j} + rand_1 * Best_j - rand_2 * Worst_j$ *// for pFOA_v2*
    endfor
    $fnew_i = fit(\overrightarrow{Snew_i})$
  endfor

  if $fnew_i < f_i$ //
    $f_i = fnew_i$ & $\overrightarrow{S_i} = \overrightarrow{Snew_i}$
  endif

endfor

FIGURE 2. Pseudo code of pFOA.

to this direction. In the vision phase, it reaches food by using sharp vision. The steps of the algorithm are as follows.

Step 1: Determine the initial position of the fruit fly swarm randomly.

$$Init\ X_{axis};\ Init\ Y_{axis} \tag{1}$$

Step 2: Smell phase. Randomly update of fruit fly locations.

$$\begin{aligned} X_i &= X_{axis} + Rand \\ Y_i &= Y_{axis} + Rand \end{aligned} \tag{2}$$

Step 3: Calculate the smell concentration decision value S_i of each fruit fly. Use the distance of the fruit fly from the origin $Dist_i$ to calculate S_i.

$$\begin{aligned} Dist_i &= \sqrt{(X_i^2 + Y_i^2)} \\ S_i &= {}^{1}/_{Dist_i} \end{aligned} \tag{3}$$

Step 4: Substitute smell concentration judgment value (S_i) into the fitness function to calculate the smell ($Smell_i$).

$$Smell_i = Function(S_i) \tag{4}$$

Step 5: Find the fruit fly with the best smell value.

$$[bestSmell\ bestIndex] = \max(Smell) \tag{5}$$

Step 6: Vision Phase. Record best smell value and x, y coordinate. If this smell value is better than the previous smell value, fruit flies will use this location for vision.

$$\begin{aligned} Smellbest &= bestSmell \\ X_{axis} &= X\ (bestIndex) \\ Y_{axis} &= Y(bestIndex \end{aligned} \tag{6}$$

Step 7: If a termination condition is met, report the best solution, otherwise go to Step 2.

III. JAYA ALGORITHM

The Jaya algorithm was proposed by Rao in 2016 [58], [59]. The algorithm always tries to reach the best solution and move away from the worst solution. To achieve this goal, the algorithm uses the following equation 7 to obtain the new solution.

$$X'_{k,j} = X_{k,j} + r_{1,j} * \left(Best_j - \left|X_{k,j}\right|\right) - r_{2,j} * \times \left(Worst_j - \left|X_{k,j}\right|\right) \tag{7}$$

where, k is candidate solution index that is processed on, Best and Worst are the best and worst solution obtained so far, respectively. j is the dimension index, r_1 and r_2 stand for random values produced in range of [0, 1].

The term "$r_{1,j}$*(Best-$|X_{k,j}|$)" indicates the tendency of the solution to move closer to the best solution.

The term "$-r_{2,j}$*(Worst-$|X_{k,j}|$)" indicates the tendency of the solution to avoid the worst solution.

After candidate solution is generated, the fitness values of the candidate and current solutions are calculated. A greedy approach is applied between current and candidate solutions.

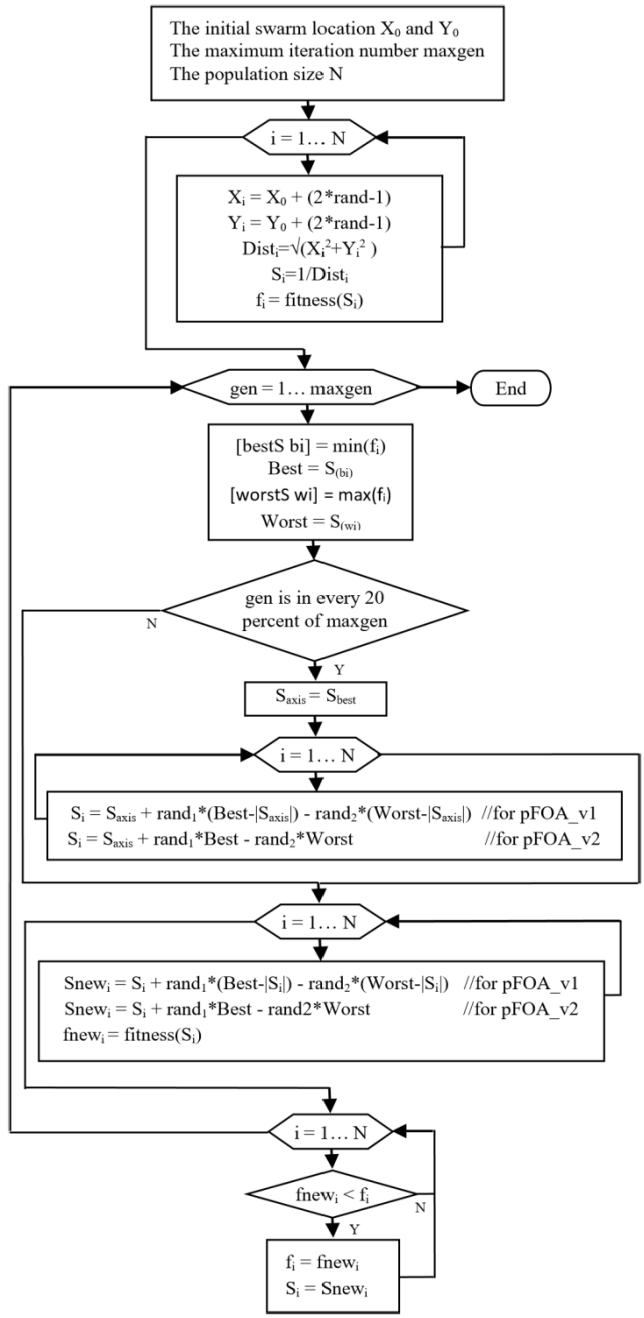

FIGURE 3. Flow chart of pFOA.

If the fitness of candidate solution is better than the fitness of the current solution, the candidate solution is replaced with the current solution in the population. The Jaya algorithm is an iterative optimization algorithm and the process given above is repeated until a termination condition is met.

IV. THE PROPOSED APPROACH

When we analyze the basic FOA, we see that all the artificial agents in the algorithm follow the best solution. Therefore, the algorithm shows a fast convergence characteristics but this can cause a stagnation in the population. When the best

TABLE 1. Benchmark functions.

No	Name	SR	Class	Function
F1	Sphere	[-100,100]	US	$f_1(\vec{X})=\sum_{i=1}^{D} x_i^2$
F2	Elliptic	[-100,100]	UN	$f_2(\vec{X})=\sum_{i=1}^{D} (10^6)^{(i-1)/(D-1)} x_i^2$
F3	SumSquares	[-10,10]	US	$f_3(\vec{X})=\sum_{i=1}^{D} i x_i^2$
F4	SumPower	[-10,10]	MS	$f_4(\vec{X})=\sum_{i=1}^{D} \lvert x_i \rvert^{(i+1)}$
F5	Schwefel2.22	[-10,10]	UN	$f_5(\vec{X})=\sum_{i=1}^{D} \lvert x_i \rvert+\prod_{i=1}^{D} \lvert x_i \rvert$
F6	Schwefel2.21	[-100,100]	UN	$f_6(\vec{X})=\max_i \{\lvert x_i \rvert, 1\le i\le D\}$
F7	Step	[-100,100]	US	$f_7(\vec{X})=\sum_{i=1}^{D} (\lfloor x_i+0.5 \rfloor)^2$
F8	Quartic	[-1.28,1.28]	US	$f_8(\vec{X})=\sum_{i=1}^{D} i x_i^4$
F9	QuarticWN	[-1.28,1.28]	US	$f_9(\vec{X})=\sum_{i=1}^{D} i x_i^4+random[0,1)$
F10	Rosenbrock	[-10,10]	UN	$f_{10}(\vec{X})=\sum_{i=1}^{D-1}\left[100\left(x_{i+1}-x_i^2\right)^2+\left(x_i-1\right)^2\right]$
F11	Rastrigin	[-5.12,5.12]	MS	$f_{11}(\vec{X})=\sum_{i=1}^{D}\left[x_i^2-10\cos(2\pi x_i)+10\right]$
F12	Non-Continuous Rastrigin	[-5.12,5.12]	MS	$f_{12}(\vec{X})=\sum_{i=1}^{D}\left[y_i^2-10\cos(2\pi y_i)+10\right]$ $y_i=\begin{cases} x_i & \lvert x_i \rvert<\frac{1}{2} \\ \frac{round(2x_i)}{2} & \lvert x_i \rvert\ge\frac{1}{2} \end{cases}$
F13	Griewank	[-600,600]	MN	$f_{13}(\vec{X})=\frac{1}{4000}\sum_{i=1}^{D} x_i^2-\prod_{i=1}^{D}\cos\left(\frac{x_i}{\sqrt{i}}\right)+1$
F14	Schwefel2.26	[-500,500]	UN	$f_{14}(\vec{X})=418.98*\mathrm{D}-\sum_{i=1}^{n} x_i \sin\left(\sqrt{\lvert x_i \rvert}\right)$

TABLE 1. ***(Continued.)*** **Benchmark functions.**

F15	Ackley	[-32,32]	MN	$f_{15}(\vec{X}) = -20\exp\left\{-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D} x_i^2}\right\} - \exp\left\{\frac{1}{D}\sum_{i=1}^{D}\cos(2\pi x_i)\right\} + 20 + e$
F16	Penalized1	[-50,50]	MN	$f_{16}(\vec{X}) = \frac{\pi}{D}\{10\sin^2(\pi y_1) + \sum_{i=1}^{D-1}(y_i - 1)^2[1 + 10\sin^2(\pi y_{i+1})] + (y_D - 1)^2\} + \sum_{i=1}^{D} u(x_i, 10, 100, 4)$ $y_i = 1 + \frac{1}{4}(x_i + 1)$ $u_{x_i,a,k,m} = \begin{cases} k(x_i - a)^m & x_i > a \\ 0 & -a \le x_i \le a \\ k(x_i - a)^m & x_i < -a \end{cases}$
F17	Penalized2	[-50,50]	MN	$f_{17}(\vec{X}) = \frac{1}{10}\{\sin^2(\pi x_1) + \sum_{i=1}^{D-1}(x_i - 1)^2[1 + \sin^2(3\pi x_{i+1})] + (x_D - 1)^2[1 + \sin^2(2\pi x_{i+1})]\} + \sum_{i=1}^{D} u(x_i, 5, 100, 4)$
F18	Alpine	[-10,10]	MS	$f_{18}(\vec{X}) = \sum_{i=1}^{D}\lvert x_i \cdot \sin(x_i) + 0.1 \cdot x_i\rvert$
F19	Levy	[-10,10]	MN	$f_{19}(\vec{X}) = \sum_{i=1}^{D-1}(x_i - 1)^2[1 + \sin^2(3\pi x_{i+1})] + \sin^2(3\pi x_1) + \lvert x_D - 1\rvert[1 + \sin^2(3\pi x_D)]$
F20	Weierstrass	[-0.5,0.5]	MN	$f_{20}(\vec{X}) = \sum_{i=1}^{D}(\sum_{k=0}^{k_{\max}}[a^k\cos(2\pi b^k(x_i + 0.5))]) - D\sum_{k=0}^{k_{\max}}[a^k\cos(2\pi b^k 0.5)]$ $a = 0.5, b = 3, k_{\max} = 20$
F21	Schaffer	[-100,100]	MN	$f_{21}(\vec{X}) = 0.5 + \frac{\sin^2\left(\sqrt{\sum_{i=1}^{D} x_i^2}\right) - 0.5}{\left(1 + 0.001 * \left[\sum_{i=1}^{D} x_i^2\right]\right)^2}$

solution in the population is a local minimum point on the search space, the fast convergence is premature convergence and the optimum solution cannot be obtained during the search with FOA. Another problem in the basic algorithm is the search only on the positive side of the solution space because the distance is always results with a positive value.

The algorithm has been modified to solve these problems and improve the performance of the proposed algorithm. While the smell phase is the same as in the basic FOA in this study, the visual phase has been modified by considering a new update rule. In the visual phase, the update strategy of the basic FOA and the JAYA algorithm has been used together. At the same time, two different formula (version1 and version2) has been used to obtain new candidate solutions. The proposed approach is called pFOA_v1 and pFOA_v2.

In pFOA_v1, new solutions are obtained by Eq. 7 which is proposed and used in Jaya algorithm.

In pFOA_v2, we used a bit modified version of Eq.7. The new candidate solutions are obtained using the following equation (Eq. 8).

$$X'_{k,j} = X_{k,j} + r_{1,j} * Best_j - r_{2,j} * Worst_j \tag{8}$$

Being considered new formulae, we restrict the solution space between the best and worst solution in the population, and the intensification capability of the pFOA_v2 has been strengthened.

The changes in the visual phase of the algorithm are as follows;

Smell concentration judgment values (S_i) are updated as follows:

TABLE 2. Comparative results of pFOA_v1 and basic FOA algorithms for 10, 30 and 50 dimensions.

		pFOA_v1		FOA			pFOA_v1		FOA			pFOA_v1		FOA	
Func.	D	Mean	Std. Dev.	Mean	Std. Dev.	D	Mean	Std. Dev.	Mean	Std. Dev.	D	Mean	Std. Dev.	Mean	Std. Dev.
F1	10	**1,46E-154**	1,00E-153	3,21E-06	1,52E-06	30	**3,41E-146**	1,23E-145	3,30E-06	1,56E-06	50	**6,63E-116**	3,07E-115	3,33E-06	1,57E-06
F2	10	**4,89E-153**	2,09E-152	1,18E-01	3,72E-02	30	**9,73E-142**	4,64E-141	8,36E-02	2,63E-02	50	**1,82E-111**	7,21E-111	7,81E-02	2,46E-02
F3	10	**1,79E-142**	8,95E-142	1,74E-05	4,35E-06	30	**8,95E-142**	2,40E-144	4,80E-05	1,20E-05	50	**2,18E-115**	6,28E-115	7,87E-05	1,96E-05
F4	10	**1,66E-273**	0,00E+00	4,06E-08	8,66E-09	30	**2,08E-227**	0,00E+00	4,43E-09	9,43E-10	50	**1,48E-161**	7,25E-161	1,58E-09	3,37E-10
F5	10	**2,53E-81**	4,99E-81	5,87E-03	1,11E-03	30	**5,46E-79**	2,47E-78	1,02E-02	1,92E-03	50	**4,15E-51**	2,90E-50	1,31E-02	2,48E-03
F6	10	**5,93E-46**	4,15E-45	1,12E-03	1,92E-04	30	**6,86E-05**	2,12E-04	2,44E-03	4,18E-04	50	1,43E-02	1,86E-03	**4,46E-03**	7,66E-04
F7	10	**0,00E+00**	0,00E+00	**0,00E+00**	0,00E+00	30	**0,00E+00**	0,00E+00	**0,00E+00**	0,00E+00	50	**0,00E+00**	0,00E+00	**0,00E+00**	0,00E+00
F8	10	**1,29E-234**	0,00E+00	5,93E-12	8,75E-13	30	**4,61E-184**	0,00E+00	5,35E-12	7,89E-13	50	**3,28E-128**	2,12E-127	5,23E-12	7,71E-13
F9	10	**9,01E-04**	4,80E-04	2,77E-03	3,93E-04	30	1,15E-01	7,50E-01	**5,78E-03**	8,09E-04	50	1,96E-01	1,13E+00	**7,82E-03**	1,09E-03
F10	10	**1,60E-03**	1,98E-03	6,99E+00	9,19E-01	30	**9,34E-09**	4,47E-08	2,75E+01	3,61E+00	50	**5,79E-01**	9,82E-01	4,80E+01	6,31E+00
F11	10	**7,22E-01**	4,48E-01	5,58E+00	1,04E+00	30	**4,76E+00**	4,30E+00	4,16E+01	6,21E+00	50	**8,23E+00**	4,02E+00	1,04E+02	1,42E+01
F12	10	**9,80E-01**	7,07E-01	1,38E+00	2,09E-01	30	**6,56E+00**	8,64E+00	1,49E+01	3,32E+00	50	**1,31E+01**	1,01E+01	7,10E+01	1,07E+01
F13	10	**0,00E+00**	0,00E+00	2,85E-07	3,28E-08	30	**8,49E-17**	1,61E-16	1,38E-07	1,58E-08	50	**8,51E-16**	7,04E-16	9,30E-08	1,07E-08
F14	10	4,15E+03	5,43E-12	**4,00E+03**	4,41E+02	30	1,25E+04	3,21E+00	**1,22E+04**	1,35E+03	50	**1,98E+04**	3,24E+03	2,04E+04	2,25E+03
F15	10	**3,69E-15**	6,96E-16	2,35E-03	2,50E-04	30	**2,31E-14**	1,16E-14	1,35E-03	1,44E-04	50	**1,90E-13**	1,86E-13	1,05E-03	1,12E-04
F16	10	**6,88E-02**	1,95E-02	2,65E+00	2,74E-01	30	**4,45E-01**	1,88E-01	1,67E+00	1,72E-01	50	**6,26E-01**	1,05E-01	1,47E+00	1,52E-01
F17	10	9,75E-01	2,56E-02	**9,54E-01**	9,57E-02	30	**2,82E+00**	4,99E-01	2,90E+00	2,90E-01	50	**4,45E+00**	1,18E+00	4,85E+00	4,85E-01
F18	10	**4,31E-05**	1,55E-04	3,00E-03	1,69E-03	30	**5,53E-31**	1,65E-30	1,62E-02	7,50E-03	50	**1,23E-52**	5,41E-52	1,19E-02	7,37E-03
F19	10	**3,37E+00**	2,81E+00	5,98E+00	5,82E-01	30	**7,23E-01**	6,77E-01	2,12E+01	2,02E+00	50	**9,50E-01**	6,87E-01	3,77E+01	3,57E+00
F20	10	3,99E+01	5,60E-01	**1,02E+01**	9,38E-01	30	1,20E+02	4,90E-01	**4,43E+01**	4,09E+00	50	2,00E+02	1,83E-01	**8,02E+01**	7,38E+00
F21	10	**0,00E+00**	0,00E+00	3,21E-06	2,89E-07	30	**1,15E-16**	4,82E-17	3,30E-06	2,97E-07	50	**3,13E-16**	1,56E-16	1,94E-04	1,23E-04

TABLE 3. Comparative results of pFOA_v2 and basic FOA algorithms for 10, 30 and 50 dimensions.

		pFOA_v2		FOA			pFOA_v2		FOA			pFOA_v2		FOA	
Func.	D	Mean	Std. Dev.	Mean	Std. Dev.	D	Mean	Std. Dev.	Mean	Std. Dev.	D	Mean	Std. Dev.	Mean	Std. Dev.
F1	10	**2,32E-158**	1,62E-157	3,21E-06	1,52E-06	30	**4,53E-197**	0,00E+00	3,30E-06	1,56E-06	50	**4,15E-59**	2,96E-58	3,33E-06	1,57E-06
F2	10	**2,09E-155**	1,05E-154	1,18E-01	3,72E-02	30	**2,97E-198**	0,00E+00	8,36E-02	2,63E-02	50	**4,14E-109**	2,09E-108	7,81E-02	2,46E-02
F3	10	**6,89E-156**	2,69E-155	1,74E-05	4,35E-06	30	**2,84E-179**	0,00E+00	4,80E-05	1,20E-05	50	**1,56E-163**	0,00E+00	7,87E-05	1,96E-05
F4	10	**2,09E-271**	0,00E+00	4,06E-08	8,66E-09	30	**2,91E-91**	1,04E-90	4,43E-09	9,43E-10	50	**2,93E-104**	1,05E-103	1,58E-09	3,37E-10
F5	10	**7,59E-78**	8,91E-78	5,87E-03	1,11E-03	30	**2,10E-50**	6,71E-50	1,02E-02	1,92E-03	50	**6,42E-65**	1,46E-64	1,31E-02	2,48E-03
F6	10	**2,01E-47**	3,48E-47	1,12E-03	1,92E-04	30	**1,38E-09**	3,16E-09	2,44E-03	4,18E-04	50	**2,56E-07**	3,94E-07	4,46E-03	7,66E-04
F7	10	**0,00E+00**	0,00E+00	**0,00E+00**	0,00E+00	30	**0,00E+00**	0,00E+00	**0,00E+00**	0,00E+00	50	**0,00E+00**	0,00E+00	**0,00E+00**	0,00E+00
F8	10	**2,38E-265**	0,00E+00	5,93E-12	8,75E-13	30	**2,27E-133**	5,73E-133	5,35E-12	7,89E-13	50	**7,54E-73**	1,90E-72	5,23E-12	7,71E-13
F9	10	**8,08E-04**	4,53E-04	2,77E-03	3,93E-04	30	**3,86E-03**	1,45E-03	5,78E-03	8,09E-04	50	**6,57E-03**	2,41E-03	7,82E-03	1,09E-03
F10	10	**5,69E+00**	2,92E-01	6,99E+00	9,19E-01	30	**2,67E+01**	8,01E+00	2,75E+01	3,61E+00	50	**4,69E+01**	1,41E+01	4,80E+01	6,31E+00
F11	10	**6,15E-01**	6,45E-01	5,58E+00	1,04E+00	30	**2,46E+00**	2,50E+00	4,16E+01	6,21E+00	50	**4,34E+00**	3,61E+00	1,04E+02	1,42E+01
F12	10	**3,49E-01**	2,61E-01	1,38E+00	2,09E-01	30	**1,30E+01**	5,37E+00	1,49E+01	3,32E+00	50	**2,40E+01**	9,76E+00	7,10E+01	1,07E+01
F13	10	**0,00E+00**	0,00E+00	2,85E-07	3,28E-08	30	**0,00E+00**	0,00E+00	1,38E-07	1,58E-08	50	**0,00E+00**	0,00E+00	9,30E-08	1,07E-08
F14	10	**3,96E+03**	1,03E+03	4,00E+03	4,41E+02	30	**8,96E+03**	2,34E+03	1,22E+04	1,35E+03	50	**1,55E+04**	4,00E+03	2,04E+04	2,25E+03
F15	10	**1,67E-15**	6,20E-16	2,35E-03	2,50E-04	30	**3,62E-15**	9,13E-16	1,35E-03	1,44E-04	50	**4,67E-15**	1,24E-15	1,05E-03	1,12E-04
F16	10	**9,15E-03**	2,39E-03	2,65E+00	2,74E-01	30	**7,56E-02**	1,88E-02	1,67E+00	1,72E-01	50	**1,64E-01**	4,02E-02	1,47E+00	1,52E-01
F17	10	9,67E-01	2,29E-01	**9,54E-01**	9,57E-02	30	**2,81E+00**	6,64E-01	2,90E+00	2,90E-01	50	**4,76E+00**	1,12E+00	4,85E+00	4,85E-01
F18	10	**1,26E-03**	5,10E-04	3,00E-03	1,69E-03	30	**3,43E-04**	1,98E-04	1,62E-02	7,50E-03	50	**1,01E-04**	1,07E-04	1,19E-02	7,37E-03
F19	10	**1,00E+00**	2,68E-01	5,98E+00	5,82E-01	30	**8,43E+00**	1,92E+00	2,12E+01	2,02E+00	50	**2,40E+01**	5,39E+00	3,77E+01	3,57E+00
F20	10	**0,00E+00**	0,00E+00	1,02E+01	9,38E-01	30	**0,00E+00**	0,00E+00	4,43E+01	4,09E+00	50	**0,00E+00**	0,00E+00	8,02E+01	7,38E+00
F21	10	**0,00E+00**	0,00E+00	3,21E-06	2,89E-07	30	**0,00E+00**	0,00E+00	3,30E-06	2,97E-07	50	**1,20E-17**	5,60E-18	1,94E-04	1,23E-04

For pFOA_v1;

$$Snew_{i,j} = S_{i,j} + rand_1 * (Best_j - |S_{i,j}|) - rand_2 * (Worst_j - |S_{i,j}|) \tag{9}$$

For pFOA_v2;

$$Snew_{i,j} = S_{i,j} + rand_1 * Best_j - rand_2 * Worst_j \tag{10}$$

The objective function value ($fnew_i$) specific for the optimization problem is obtained by considering the new smell concentration judgment value ($Snew_i$) as follows:

$$f\,new_i = fit\left(\overrightarrow{Snew_i}\right) \tag{11}$$

A greedy selection is applied to the new and current candidate solutions. If the new smell value is better than the current smell value, the new smell and smell concentration judgment values (S_i) of each fruit fly are memorized. This procedure is given as follows:

$$f\,new_i < f_i \quad f_i = f\,new_i \;\&\; \overrightarrow{S_i} = \overrightarrow{Snew_i} \tag{12}$$

In every twenty percent of the maximum iteration, the fruit fly with the best smell value is fixed by using Eq. 13. The best smell concentration judgment value is assumed to be the initial value, and the fruit flies are reproduced using

TABLE 4. Comparative results of pFOA_v1 and pFOA_v2 algorithms for 10, 30 and 50 dimensions.

		pFOA_v1		pFOA_v2			pFOA_v1		pFOA_v2			pFOA_v1		pFOA_v2	
Func.	D	Mean	Std. Dev.	Mean	Std. Dev.	D	Mean	Std. Dev.	Mean	Std. Dev.	D	Mean	Std. Dev.	Mean	Std. Dev.
F1	10	1,46E-154	1,00E-153	**2,32E-158**	1,62E-157	30	3,41E-146	1,23E-145	**4,53E-197**	0,00E+00	50	6,63E-116	3,07E-115	**1,97E-164**	0,00E+00
F2	10	4,89E-153	2,09E-152	**2,09E-155**	1,05E-154	30	9,73E-142	4,64E-141	**2,97E-198**	0,00E+00	50	1,82E-111	7,21E-111	**3,50E-228**	0,00E+00
F3	10	1,79E-142	8,95E-142	**6,89E-156**	2,69E-155	30	8,95E-142	2,40E-144	**2,84E-179**	0,00E+00	50	2,18E-115	6,28E-115	**1,56E-163**	0,00E+00
F4	10	**1,66E-273**	0,00E+00	2,09E-271	0,00E+00	30	2,08E-227	0,00E+00	**3,02E-257**	0,00E+00	50	1,48E-161	7,25E-161	**6,88E-236**	0,00E+00
F5	10	**2,53E-81**	4,99E-81	7,59E-78	8,91E-78	30	5,46E-79	2,47E-78	**1,39E-97**	9,91E-97	50	4,15E-51	2,90E-50	**7,93E-137**	5,66E-136
F6	10	5,93E-46	4,15E-45	**2,01E-47**	3,48E-47	30	6,86E-05	2,12E-04	**1,38E-09**	3,16E-09	50	1,43E-02	1,86E-03	**2,56E-07**	3,94E-07
F7	10	**0,00E+00**	0,00E+00	**0,00E+00**	0,00E+00	30	**0,00E+00**	0,00E+00	**0,00E+00**	0,00E+00	50	**0,00E+00**	0,00E+00	**0,00E+00**	0,00E+00
F8	10	1,29E-234	0,00E+00	**2,38E-265**	0,00E+00	30	4,61E-184	0,00E+00	**5,80E-258**	0,00E+00	50	3,28E-128	2,12E-127	**7,75E-180**	0,00E+00
F9	10	9,01E-04	4,80E-04	**8,08E-04**	4,53E-04	30	1,15E-01	7,50E-01	**3,86E-03**	1,45E-03	50	1,96E-01	1,13E+00	**5,05E-03**	3,22E-03
F10	10	**1,60E-03**	1,98E-03	5,69E+00	2,92E-01	30	**9,34E-09**	4,47E-08	2,67E+01	2,95E-01	50	**5,79E-01**	9,82E-01	4,69E+01	1,41E+01
F11	10	7,22E-01	4,48E-01	**6,15E-01**	6,45E-01	30	4,76E+00	4,30E+00	**2,46E+00**	2,50E+00	50	8,23E+00	4,02E+00	**4,34E+00**	3,61E+00
F12	10	9,80E-01	7,07E-01	**3,49E-01**	2,61E-01	30	**6,56E+00**	8,64E+00	1,30E+01	5,37E+00	50	**1,31E+01**	1,01E+01	2,40E+01	9,76E+00
F13	10	**0,00E+00**	0,00E+00	**0,00E+00**	0,00E+00	30	8,49E-17	1,61E-16	**0,00E+00**	0,00E+00	50	8,51E-16	7,04E-16	**0,00E+00**	0,00E+00
F14	10	4,15E+03	5,43E-12	**3,96E+03**	1,03E+03	30	1,25E+04	3,21E+00	**8,96E+03**	2,34E+03	50	1,98E+04	3,24E+03	**1,55E+04**	4,00E+03
F15	10	3,69E-15	6,96E-16	**1,67E-15**	6,20E-16	30	2,31E-14	1,16E-14	**3,62E-15**	9,13E-16	50	1,90E-13	1,86E-13	**4,67E-15**	1,24E-15
F16	10	6,88E-02	1,95E-02	**9,15E-03**	2,39E-03	30	4,45E-01	1,88E-01	**7,56E-02**	1,88E-02	50	6,26E-01	1,05E-01	**1,52E-01**	2,29E-02
F17	10	9,75E-01	2,56E-02	**9,67E-01**	2,29E-01	30	2,82E+00	4,99E-01	**2,81E+00**	6,64E-01	50	**4,45E+00**	1,18E+00	4,73E+00	1,88E-01
F18	10	**4,31E-05**	1,55E-04	1,26E-03	5,10E-04	30	**5,53E-31**	1,65E-30	3,43E-04	1,98E-04	50	**1,23E-52**	5,41E-52	2,25E-05	7,15E-05
F19	10	3,37E+00	2,81E+00	**1,00E+00**	2,68E-01	30	**7,23E-01**	6,77E-01	8,43E+00	1,92E+00	50	**9,50E-01**	6,87E-01	2,40E+01	5,39E+00
F20	10	3,99E+01	5,60E-01	**0,00E+00**	0,00E+00	30	1,20E+02	4,90E-01	**0,00E+00**	0,00E+00	50	2,00E+02	1,83E-01	**0,00E+00**	0,00E+00
F21	10	**0,00E+00**	0,00E+00	**0,00E+00**	0,00E+00	30	1,15E-16	4,82E-17	**0,00E+00**	0,00E+00	50	3,13E-16	1,56E-16	**1,20E-17**	5,60E-18

TABLE 5. Comparative results of pFOA versions, JAYA and basic FOA algorithms for 10 dimension.

		pFOA_v1		pFOA_v2		JAYA		FOA	
Func.	D	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
F1	10	1,46E-154	1,00E-153	**2,32E-158**	1,62E-157	1,99E-151	7,85E-151	3,21E-06	1,52E-06
F2	10	4,89E-153	2,09E-152	**2,09E-155**	1,05E-154	5,10E-148	6,63E-148	1,18E-01	3,72E-02
F3	10	1,79E-142	8,95E-142	**6,89E-156**	2,69E-155	6,44E-152	5,21E-152	1,74E-05	4,35E-06
F4	10	**1,66E-273**	0,00E+00	2,09E-271	0,00E+00	8,63E-267	0,00E+00	4,06E-08	8,66E-09
F5	10	**2,53E-81**	4,99E-81	7,59E-78	8,91E-78	3,50E-79	2,69E-79	5,87E-03	1,11E-03
F6	10	5,93E-46	4,15E-45	**2,01E-47**	3,48E-47	6,28E-45	3,77E-45	1,12E-03	1,92E-04
F7	10	**0,00E+00**	0,00E+00	**0,00E+00**	0,00E+00	**0,00E+00**	0,00E+00	**0,00E+00**	0,00E+00
F8	10	1,29E-234	0,00E+00	**2,38E-265**	0,00E+00	4,46E-249	0,00E+00	5,93E-12	8,75E-13
F9	10	9,01E-04	4,80E-04	**8,08E-04**	4,53E-04	1,08E-03	2,18E-04	2,77E-03	3,93E-04
F10	10	1,60E-03	1,98E-03	5,69E+00	2,92E-01	**1,10E-03**	1,05E-03	6,99E+00	9,19E-01
F11	10	7,22E-01	4,48E-01	**6,15E-01**	6,45E-01	3,81E+00	7,77E-01	5,58E+00	1,04E+00
F12	10	9,80E-01	7,07E-01	**3,49E-01**	2,61E-01	5,77E+00	9,95E-01	1,38E+00	2,09E-01
F13	10	**0,00E+00**	0,00E+00	**0,00E+00**	0,00E+00	**0,00E+00**	0,00E+00	2,85E-07	3,28E-08
F14	10	4,15E+03	5,43E-12	**3,96E+03**	1,03E+03	4,15E+03	6,33E+02	4,00E+03	4,41E+02
F15	10	3,69E-15	6,96E-16	**1,67E-15**	6,20E-16	3,76E-15	5,68E-16	2,35E-03	2,50E-04
F16	10	6,88E-02	1,95E-02	**9,15E-03**	2,39E-03	2,07E-02	3,11E-03	2,65E+00	2,74E-01
F17	10	9,75E-01	2,56E-02	9,67E-01	2,29E-01	**2,15E-04**	2,15E-04	9,54E-01	9,57E-02
F18	10	**4,31E-05**	1,55E-04	1,26E-03	5,10E-04	5,29E-05	4,51E-05	3,00E-03	1,69E-03
F19	10	3,37E+00	2,81E+00	1,00E+00	2,68E-01	**2,15E-03**	2,04E-03	5,98E+00	5,82E-01
F20	10	3,99E+01	5,60E-01	**0,00E+00**	0,00E+00	1,04E+01	1,61E+00	1,02E+01	9,38E-01
F21	10	**0,00E+00**	0,00E+00	**0,00E+00**	0,00E+00	9,72E-03	1,21E-03	3,21E-06	2,89E-07

the by the equation Eq. 14 or Eq. 15 by depending on the version.

$$\overrightarrow{S_{axis}} = \overrightarrow{S_{best}} \tag{13}$$

For pFOA_v1;

$$S_{i,j} = S_{axis,j} + rand_1 * (Best_j - |S_{axis,j}|) - rand_2 * (Worst_j - |S_{axis,j}|) \tag{14}$$

For pFOA_v2;

$$S_{i,j} = S_{axis,j} + rand_1 * Best_j - rand_2 * Worst_j \tag{15}$$

This modification makes two major contributions to the basic FOA. The first is to evaluate the best and worst solutions and to achieve the real solution. This improves the global search ability of the basic algorithm and the solution space is effectively searched with FOA. Second, the fitness function can take a negative value. Thus, the developed algorithm can solve problems involving negative solutions. This allows the

TABLE 6. Comparative results of pFOA versions, JAYA and basic FOA algorithms for 30 dimension.

		pFOA_v1		pFOA_v2		JAYA		FOA	
Func.	D	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
F1	30	3,41E-146	1,23E-145	**4,53E-197**	0,00E+00	9,97E-146	2,13E-145	3,30E-06	1,56E-06
F2	30	9,73E-142	4,64E-141	**2,97E-198**	0,00E+00	5,83E-141	1,49E-140	8,36E-02	2,63E-02
F3	30	8,95E-142	2,40E-144	**2,84E-179**	0,00E+00	2,56E-144	3,51E-144	4,80E-05	1,20E-05
F4	30	2,08E-227	0,00E+00	**3,02E-257**	0,00E+00	1,69E-223	0,00E+00	4,43E-09	9,43E-10
F5	30	5,46E-79	2,47E-78	**1,39E-97**	9,91E-97	1,08E-81	1,15E-81	1,02E-02	1,92E-03
F6	30	6,86E-05	2,12E-04	**1,38E-09**	3,16E-09	1,81E-04	6,62E-05	2,44E-03	4,18E-04
F7	30	**0,00E+00**	0,00E+00	**0,00E+00**	0,00E+00	1,37E-01	8,06E-02	**0,00E+00**	0,00E+00
F8	30	4,61E-184	0,00E+00	**5,80E-258**	0,00E+00	4,15E-183	0,00E+00	5,35E-12	7,89E-13
F9	30	1,15E-01	7,50E-01	**3,86E-03**	1,45E-03	9,21E-03	2,28E-03	5,78E-03	8,09E-04
F10	30	9,34E-09	4,47E-08	2,67E+01	2,95E-01	**1,17E-09**	8,47E-10	2,75E+01	3,61E+00
F11	30	4,76E+00	4,30E+00	**2,46E+00**	2,50E+00	2,03E+01	3,61E+00	4,16E+01	6,21E+00
F12	30	**6,56E+00**	8,64E+00	1,30E+01	5,37E+00	2,94E+01	5,46E+00	1,49E+01	3,32E+00
F13	30	8,49E-17	1,61E-16	**0,00E+00**	0,00E+00	4,79E-17	1,61E-17	1,38E-07	1,58E-08
F14	30	1,25E+04	3,21E+00	**8,96E+03**	2,34E+03	1,05E+04	1,72E+03	1,22E+04	1,35E+03
F15	30	2,31E-14	1,16E-14	**3,62E-15**	9,13E-16	2,22E-01	7,60E-02	1,35E-03	1,44E-04
F16	30	4,45E-01	1,88E-01	**7,56E-02**	1,88E-02	2,71E-01	3,99E-02	1,67E+00	1,72E-01
F17	30	2,82E+00	4,99E-01	2,81E+00	6,64E-01	**2,98E-03**	8,95E-04	2,90E+00	2,90E-01
F18	30	**5,53E-31**	1,65E-30	3,43E-04	1,98E-04	1,20E-17	1,16E-17	1,62E-02	7,50E-03
F19	30	7,23E-01	6,77E-01	8,43E+00	1,92E+00	**4,31E-02**	1,35E-02	2,12E+01	2,02E+00
F20	30	1,20E+02	4,90E-01	**0,00E+00**	0,00E+00	5,15E+01	6,60E+00	4,43E+01	4,09E+00
F21	30	1,15E-16	4,82E-17	**0,00E+00**	0,00E+00	9,72E-03	1,21E-03	3,30E-06	2,97E-07

TABLE 7. Comparative results of pFOA versions, JAYA and basic FOA algorithms for 50 dimension.

		pFOA_v1		pFOA_v2		JAYA		FOA	
Func.	D	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
F1	50	6,63E-116	3,07E-115	**1,97E-164**	0,00E+00	4,79E-116	1,36E-115	3,33E-06	1,57E-06
F2	50	1,82E-111	7,21E-111	**3,50E-228**	0,00E+00	3,62E-110	6,13E-110	7,81E-02	2,46E-02
F3	50	2,18E-115	6,28E-115	**1,56E-163**	0,00E+00	4,15E-114	3,92E-114	7,87E-05	1,96E-05
F4	50	1,48E-161	7,25E-161	**6,88E-236**	0,00E+00	4,38E-152	6,15E-152	1,58E-09	3,37E-10
F5	50	4,15E-51	2,90E-50	**7,93E-137**	5,66E-136	3,43E-50	6,30E-50	1,31E-02	2,48E-03
F6	50	1,43E-02	1,86E-03	**2,56E-07**	3,94E-07	5,25E-01	1,21E-01	4,46E-03	7,66E-04
F7	50	**0,00E+00**	0,00E+00	**0,00E+00**	0,00E+00	1,41E+00	3,81E-01	**0,00E+00**	0,00E+00
F8	50	3,28E-128	2,12E-127	**7,75E-180**	0,00E+00	4,69E-130	6,35E-130	5,23E-12	7,71E-13
F9	50	1,96E-01	1,13E+00	**5,05E-03**	3,22E-03	3,05E-02	6,54E-03	7,82E-03	1,09E-03
F10	50	5,79E-01	9,82E-01	4,69E+01	1,41E+01	**1,01E-01**	5,21E-02	4,80E+01	6,31E+00
F11	50	8,23E+00	4,02E+00	**4,34E+00**	3,61E+00	3,48E+01	6,11E+00	1,04E+02	1,42E+01
F12	50	**1,31E+01**	1,01E+01	2,40E+01	9,76E+00	5,29E+01	9,99E+00	7,10E+01	1,07E+01
F13	50	8,51E-16	7,04E-16	**0,00E+00**	0,00E+00	1,08E-15	3,41E-16	9,30E-08	1,07E-08
F14	50	1,98E+04	3,24E+03	1,55E+04	4,00E+03	**4,74E+03**	9,15E+02	2,04E+04	2,25E+03
F15	50	1,90E-13	1,86E-13	**4,67E-15**	1,24E-15	7,10E-01	1,38E-01	1,05E-03	1,12E-04
F16	50	6,26E-01	1,05E-01	**1,52E-01**	2,29E-02	6,35E-01	9,26E-02	1,47E+00	1,52E-01
F17	50	4,45E+00	1,18E+00	4,73E+00	1,88E-01	**9,20E-03**	2,16E-03	4,85E+00	4,85E-01
F18	50	**1,23E-52**	5,41E-52	2,25E-05	7,15E-05	2,83E-17	1,97E-17	1,19E-02	7,37E-03
F19	50	9,50E-01	6,87E-01	2,40E+01	5,39E+00	**1,50E-01**	3,72E-02	3,77E+01	3,57E+00
F20	50	2,00E+02	1,83E-01	**0,00E+00**	0,00E+00	8,90E+01	1,14E+01	8,02E+01	7,38E+00
F21	50	3,13E-16	1,56E-16	**1,20E-17**	5,60E-18	1,08E-02	1,51E-03	1,94E-04	1,23E-04

improved pFOAs to be used to solve different problems than the basic algorithm.

After these explanations, the pseudo-code of the proposed pFOA versions are given in Fig. 2.

As seen from Fig. 2, the modification is given in vision phase and the rest of the algorithm is the same with the basic FOA. Therefore, it can be easily applied to solve optimization problems as the basic version.

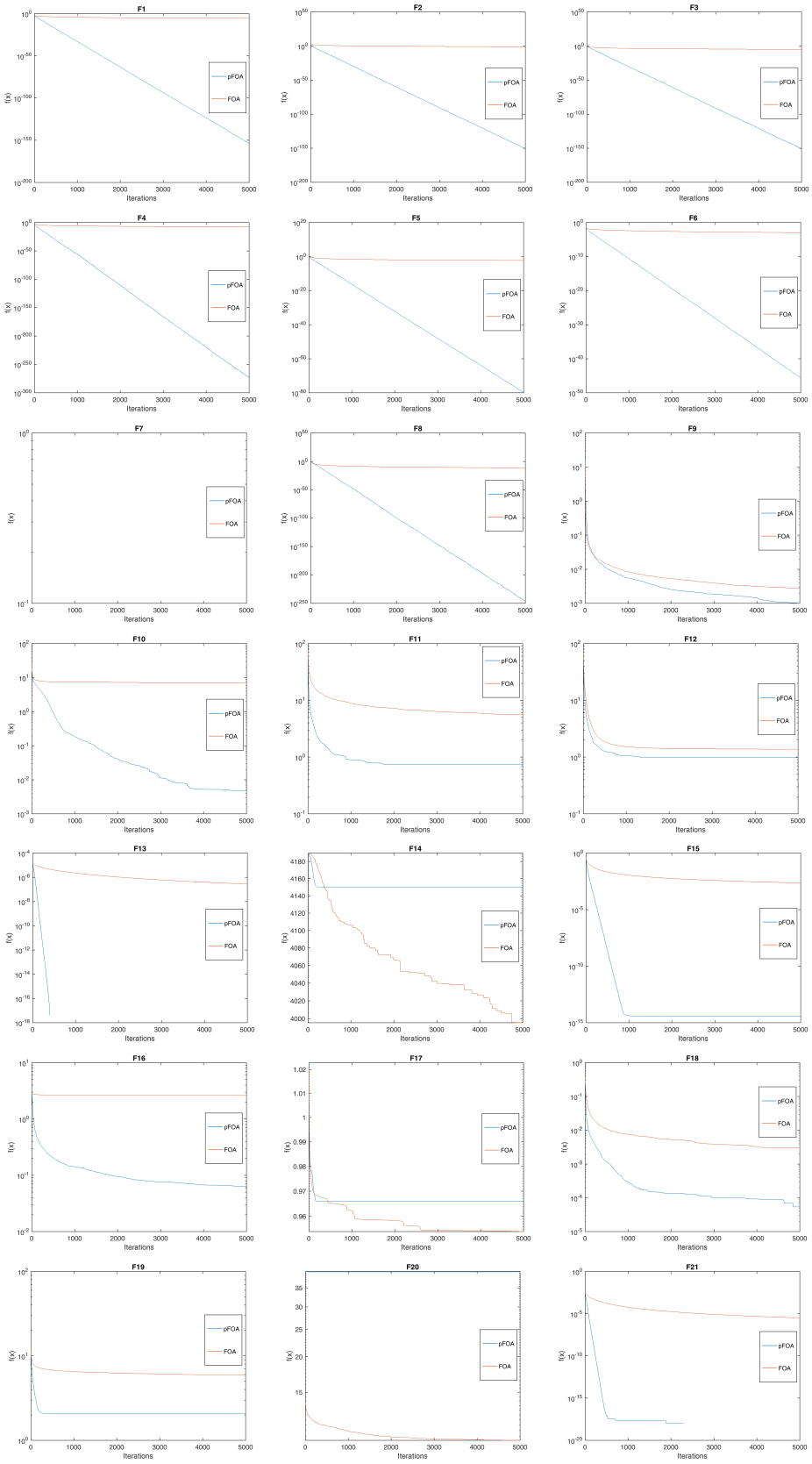

FIGURE 4. Convergence curves for 10 dimensions of pFOA_v1 and basic FOA.

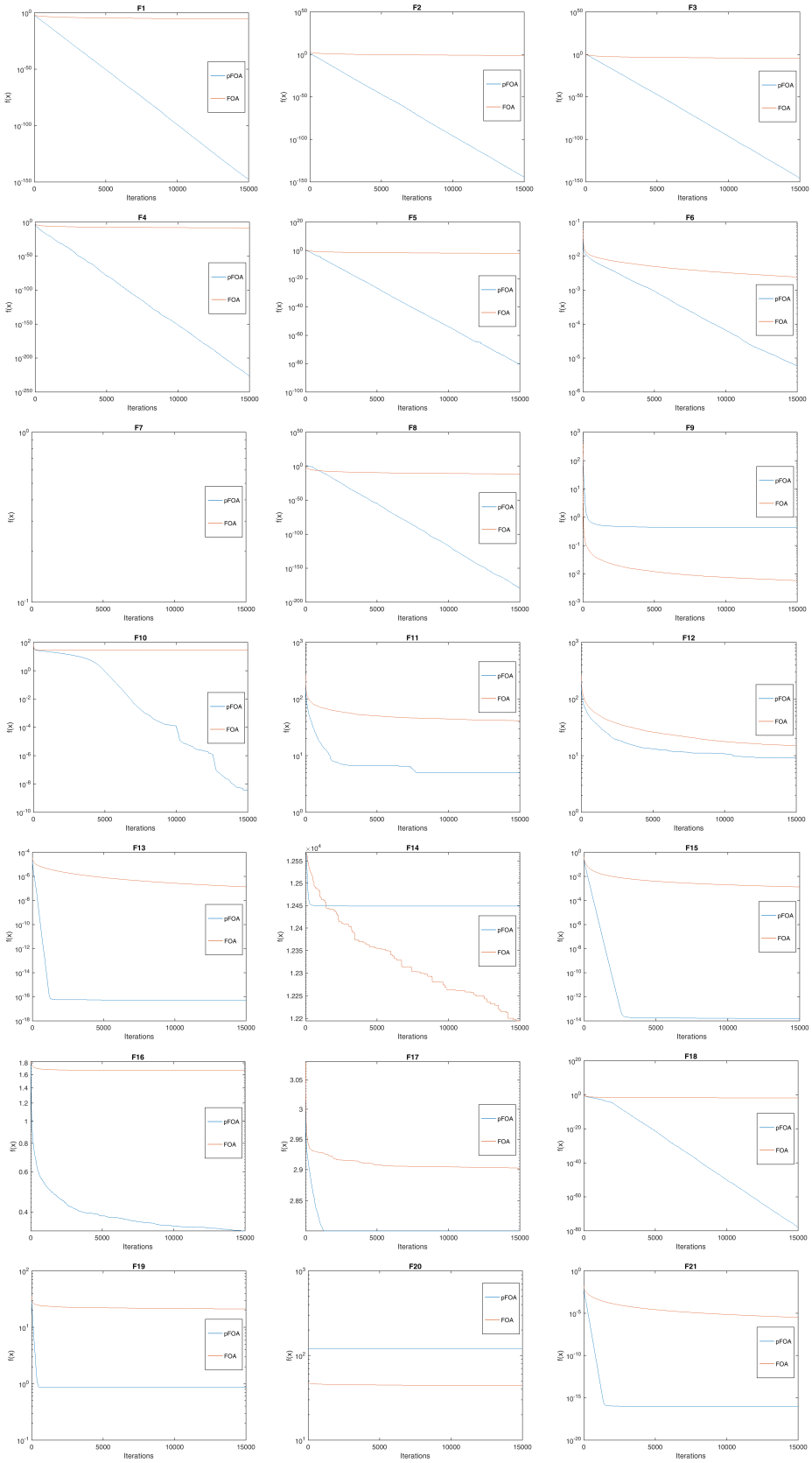

FIGURE 5. Convergence curves for 30 dimensions of pFOA_v1 and basic FOA.

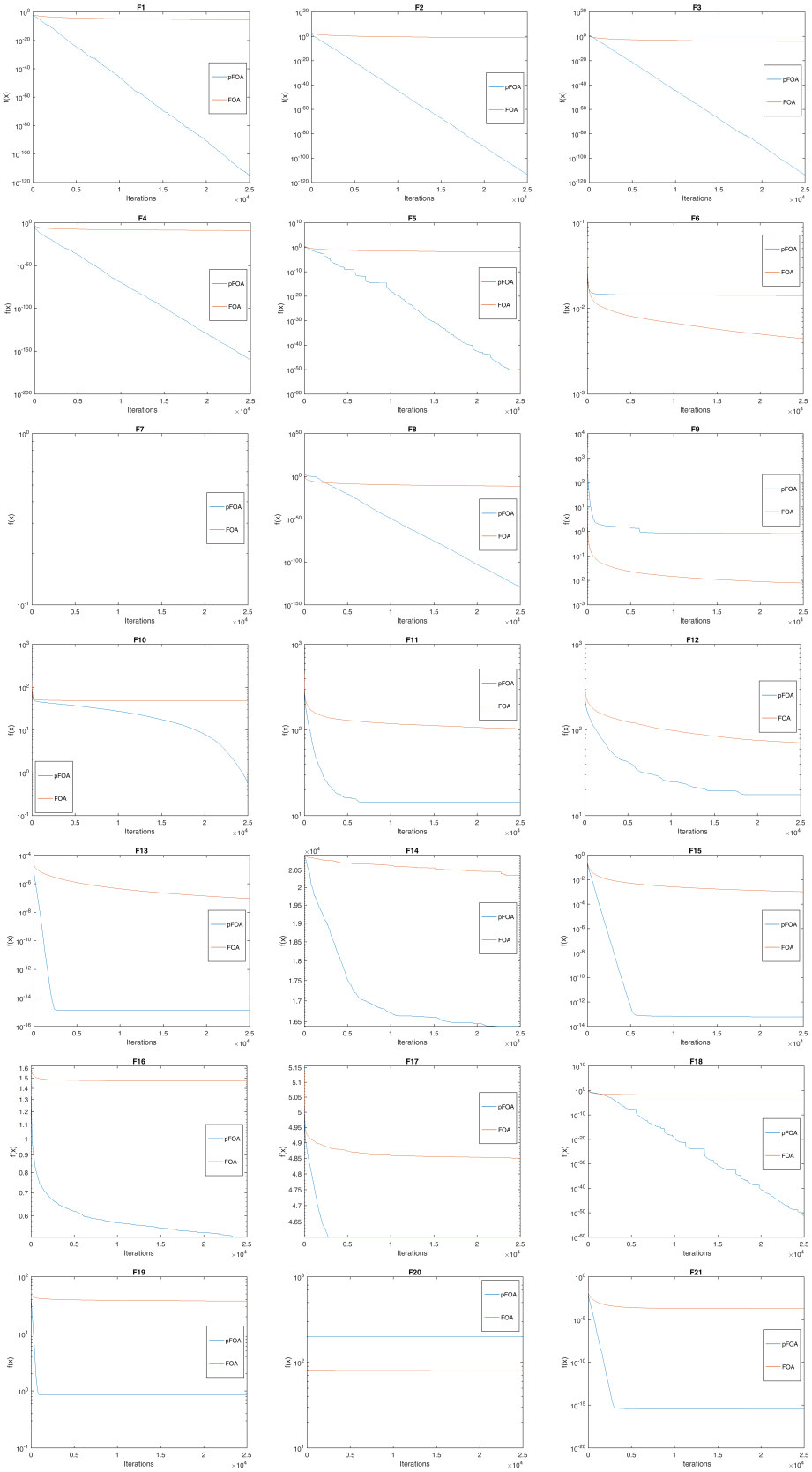

FIGURE 6. Convergence curves for 50 dimensions of pFOA_v1 and basic FOA.

TABLE 8. Comparative results of pFOA_v1 with other algorithms for 10 dimensions.

		pFOA_v1			SFOA			FOA			SPSO2011			FA			TSA			CS			JAYA		
Func.	D	Mean	Std. Dev.	R	Mean	Std. Dev.	R	Mean	Std. Dev.	R	Mean	Std. Dev.	R	Mean	Std. Dev.	R	Mean	Std. Dev.	R	Mean	Std. Dev.	R	Mean	Std. Dev.	R
F1	10	1,46E-154	1,00E-153	2	5,05E-07	2,54E-07	7	3,21E-06	1,52E-06	8	**2,50E-280**	0,00E+00	1	7,01E-88	2,70E-88	6	2,89E-109	8,34E-109	4	1,09E-90	2,49E-90	5	1,99E-151	7,85E-151	3
F2	10	**4,89E-153**	2,09E-152	1	4,19E-02	1,39E-02	6	1,18E-01	3,72E-02	7	9,98E+03	3,66E+03	8	1,89E-83	5,81E-84	5	1,24E-106	3,23E-106	3	5,81E-88	4,72E-88	4	5,10E-148	6,63E-148	2
F3	10	1,79E-142	8,95E-142	3	2,75E-06	7,09E-07	7	1,74E-05	4,35E-06	8	**3,21E-188**	0,00E+00	1	3,43E-89	8,16E-90	6	1,15E-110	2,59E-110	4	5,59E-92	5,85E-92	5	6,44E-152	5,21E-152	2
F4	10	**1,66E-273**	0,00E+00	1	2,68E-08	5,85E-09	7	4,06E-08	8,66E-09	8	5,22E-09	1,17E-09	6	6,31E-23	9,19E-23	5	4,07E-169	0,00E+00	3	8,20E-140	1,25E-139	4	8,63E-267	0,00E+00	2
F5	10	**2,53E-81**	4,99E-81	1	2,28E-03	4,38E-04	7	5,87E-03	1,11E-03	8	1,93E-71	2,55E-71	3	6,61E-45	1,20E-45	5	3,40E-69	2,35E-69	4	7,53E-42	2,15E-42	6	3,50E-79	2,69E-79	2
F6	10	5,93E-46	4,15E-45	2	2,44E-04	4,25E-05	7	1,12E-03	1,92E-04	8	**1,27E-116**	1,45E-116	1	1,51E-44	2,49E-45	4	9,54E-20	8,23E-20	6	3,17E-26	1,60E-26	5	6,28E-45	3,77E-45	3
F7	10	**0,00E+00**	0,00E+00	1	**0,00E+00**	0,00E+00	1	**0,00E+00**	0,00E+00	1	**0,00E+00**	0,00E+00	1	1,57E-01	6,82E-02	8	**0,00E+00**	0,00E+00	1	**0,00E+00**	0,00E+00	1	**0,00E+00**	0,00E+00	1
F8	10	1,29E-234	0,00E+00	3	1,40E-13	2,08E-14	7	5,93E-12	8,75E-13	8	**0,00E+00**	0,00E+00	1	1,63E-182	0,00E+00	4	1,66E-161	2,41E-161	6	8,92E-170	0,00E+00	5	4,46E-249	0,00E+00	2
F9	10	9,01E-04	4,80E-04	4	2,52E-04	3,67E-05	2	2,77E-03	3,93E-04	8	2,55E-04	3,94E-05	3	**1,31E-04**	2,13E-05	1	1,03E-03	3,49E-04	6	1,01E-03	1,55E-04	5	1,08E-03	2,18E-04	7
F10	10	1,60E-03	1,98E-03	3	8,90E+00	1,18E+00	8	6,99E+00	9,19E-01	7	2,56E-01	1,27E-01	4	1,05E+00	2,55E-01	5	1,83E+00	5,96E-01	6	**2,55E-19**	2,39E-19	1	1,10E-03	1,05E-03	2
F11	10	7,22E-01	4,48E-01	3	1,03E-04	1,30E-05	2	5,58E+00	1,04E+00	6	8,92E+00	1,40E+00	7	1,20E+01	1,64E+00	8	1,81E+00	7,33E-01	4	**0,00E+00**	0,00E+00	1	3,81E+00	7,77E-01	5
F12	10	9,80E-01	7,07E-01	2	6,55E+00	1,89E+00	6	1,38E+00	2,09E-01	3	7,10E+00	8,59E-01	7	1,20E+01	1,56E+00	8	5,97E+00	1,71E+00	5	**2,94E-01**	7,28E-02	1	5,77E+00	9,95E-01	4
F13	10	**0,00E+00**	0,00E+00	1	5,99E-08	6,92E-09	3	2,85E-07	3,28E-08	4	4,16E-02	5,46E-03	6	6,39E-02	8,14E-03	8	4,60E-02	1,81E-02	7	2,82E-03	5,94E-04	5	**0,00E+00**	0,00E+00	1
F14	10	4,15E+03	5,43E-12	6	4,19E+03	4,66E+02	8	4,00E+03	4,41E+02	5	1,04E+03	1,17E+02	4	9,06E+02	1,02E+02	3	1,16E+01	9,88E+00	2	**1,62E-11**	1,28E-11	1	4,15E+03	6,33E+02	6
F15	10	3,69E-15	6,96E-16	3	9,10E-04	9,76E-05	6	2,35E-03	2,50E-04	7	2,26E-02	1,71E-02	8	8,43E-15	9,72E-16	5	3,55E-15	8,87E-16	2	**3,34E-15**	3,65E-16	1	3,76E-15	5,68E-16	4
F16	10	6,88E-02	1,95E-02	6	2,48E+00	2,57E-01	7	2,65E+00	2,74E-01	8	6,10E-03	4,47E-03	3	1,83E-02	7,70E-03	4	**4,71E-32**	1,14E-32	1	**4,71E-32**	4,83E-33	1	2,07E-02	3,11E-03	5
F17	10	9,75E-01	2,56E-02	7	9,89E-01	9,94E-02	8	9,54E-01	9,57E-02	6	6,46E-04	2,65E-04	5	**1,35E-32**	1,33E-33	1	**1,35E-32**	3,18E-33	1	**1,35E-32**	1,34E-33	1	2,15E-04	2,15E-04	4
F18	10	4,31E-05	1,55E-04	3	1,33E-02	1,50E-03	7	3,00E-03	1,69E-03	5	5,49E-03	1,67E-03	6	**5,13E-16**	8,57E-17	1	7,93E-13	1,33E-12	2	2,36E-02	3,59E-03	8	5,29E-05	4,51E-05	4
F19	10	3,37E+00	2,81E+00	6	1,00E+01	9,53E-01	8	5,98E+00	5,82E-01	7	3,80E-02	1,99E-02	5	2,15E-03	1,44E-03	3	**1,35E-31**	3,02E-32	1	**1,35E-31**	1,27E-32	1	2,15E-03	2,04E-03	3
F20	10	3,99E+01	5,60E-01	8	1,39E+01	1,29E+00	7	1,02E+01	9,38E-01	5	3,48E-01	5,22E-02	4	4,88E-16	1,36E-16	3	**0,00E+00**	0,00E+00	1	**0,00E+00**	0,00E+00	1	1,04E+01	1,61E+00	6
F21	10	**0,00E+00**	0,00E+00	1	5,05E-07	4,56E-08	2	3,21E-06	2,89E-07	3	1,03E-02	9,75E-04	7	1,03E-02	9,71E-04	8	9,72E-03	2,07E-03	5	9,72E-03	8,69E-04	6	9,72E-03	1,21E-03	4
		Mean Rank		**3,19**			5,86			6,19			4,33			4,81			3,52			3,24			3,43
		Final Rank		**1**			7			8			5			6			4			2			3

TABLE 9. Comparative results of pFOA_v1 with other algorithms for 30 dimensions.

		pFOA_v1			SFOA			FOA			SPSO2011			FA			TSA			CS			JAYA		
Func.	D	Mean	Std. Dev.	R	Mean	Std. Dev.	R	Mean	Std. Dev.	R	Mean	Std. Dev.	R	Mean	Std. Dev.	R	Mean	Std. Dev.	R	Mean	Std. Dev.	R	Mean	Std. Dev.	R
F1	30	3,41E-146	1,23E-145	3	2,03E-07	1,02E-07	7	3,30E-06	1,56E-06	8	**1,29E-303**	0,00E+00	1	4,99E-262	0,00E+00	2	1,65E-56	2,72E-56	6	3,79E-94	1,12E-93	5	9,97E-146	2,13E-145	4
F2	30	9,73E-142	4,64E-141	2	1,31E-02	4,34E-03	6	8,36E-02	2,63E-02	7	3,96E+04	1,28E+04	8	**8,13E-257**	0,00E+00	1	2,99E-54	3,64E-54	5	8,64E-92	6,38E-92	4	5,83E-141	1,49E-140	3
F3	30	8,95E-142	2,40E-144	3	3,09E-06	7,96E-07	7	4,80E-05	1,20E-05	8	4,30E-41	2,04E-41	6	**6,92E-263**	0,00E+00	1	1,81E-57	2,00E-57	5	3,09E-96	1,89E-96	4	2,56E-144	3,51E-144	2
F4	30	**2,08E-227**	0,00E+00	1	2,99E-09	6,51E-10	5	4,43E-09	9,43E-10	6	1,02E-08	2,16E-09	7	1,12E-16	1,02E-16	4	4,70E-42	1,20E-41	3	1,00E+10	2,08E+09	8	1,69E-223	0,00E+00	2
F5	30	5,46E-79	2,47E-78	3	2,48E-03	4,77E-04	6	1,02E-02	1,92E-03	7	3,29E+00	6,95E-01	8	**9,38E-132**	1,69E-132	1	1,14E-42	6,51E-43	5	3,02E-52	8,49E-53	4	1,08E-81	1,15E-81	2
F6	30	**6,86E-05**	2,12E-04	1	8,92E-05	1,55E-05	2	2,44E-03	4,18E-04	4	7,25E+00	1,31E+00	6	1,25E+01	2,18E+00	8	6,28E-01	2,51E-01	5	8,78E+00	1,61E+00	7	1,81E-04	6,62E-05	3
F7	30	**0,00E+00**	0,00E+00	1	**0,00E+00**	0,00E+00	1	**0,00E+00**	0,00E+00	1	4,84E+00	9,77E-01	8	2,94E-01	9,88E-02	7	**0,00E+00**	0,00E+00	1	**0,00E+00**	0,00E+00	1	1,37E-01	8,06E-02	6
F8	30	4,61E-184	0,00E+00	3	2,07E-14	3,09E-15	7	5,35E-12	7,89E-13	8	**0,00E+00**	0,00E+00	1	**0,00E+00**	0,00E+00	1	4,60E-55	8,18E-55	6	1,91E-107	1,29E-107	5	4,15E-183	0,00E+00	4
F9	30	1,15E-01	7,50E-01	8	**2,35E-04**	3,36E-05	1	5,78E-03	8,09E-04	4	1,53E-03	2,31E-04	2	2,09E-03	3,77E-04	3	8,42E-03	2,75E-03	5	1,17E-02	1,78E-03	7	9,21E-03	2,28E-03	6
F10	30	9,34E-09	4,47E-08	2	2,87E+01	3,81E+00	8	2,75E+01	3,61E+00	7	1,84E+01	2,95E+00	5	1,76E+01	3,47E+00	4	2,15E+01	6,88E+00	6	6,25E-01	2,07E-01	3	**1,17E-09**	8,47E-10	1
F11	30	4,76E+00	4,30E+00	3	**4,08E-05**	5,14E-06	1	4,16E+01	6,21E+00	7	3,35E+01	4,29E+00	6	7,54E+01	9,61E+00	8	2,68E+01	1,09E+01	5	4,19E+00	6,48E-01	2	2,03E+01	3,61E+00	4
F12	30	6,56E+00	8,64E+00	3	**4,08E-05**	4,91E-06	1	1,49E+01	3,32E+00	4	4,59E+01	5,73E+00	6	1,04E+02	1,26E+01	8	6,03E+01	1,85E+01	7	5,06E+00	6,47E-01	2	2,94E+01	5,46E+00	5
F13	30	8,49E-17	1,61E-16	2	1,16E-08	1,34E-09	3	1,38E-07	1,58E-08	4	9,75E-03	1,56E-03	8	4,83E-03	9,02E-04	7	1,45E-04	2,87E-04	5	2,13E-03	6,74E-04	6	**4,79E-17**	1,61E-17	1
F14	30	1,25E+04	3,21E+00	7	1,26E+04	1,40E+03	8	1,22E+04	1,35E+03	6	4,89E+03	5,41E+02	4	4,16E+03	4,58E+02	3	1,30E+03	3,48E+02	2	**3,13E+02**	4,27E+01	1	1,05E+04	1,72E+03	5
F15	30	2,31E-14	1,16E-14	2	3,31E-04	3,55E-05	4	1,35E-03	1,44E-04	5	2,31E+00	2,53E-01	8	2,48E-14	2,78E-15	3	**6,41E-15**	1,64E-15	1	7,41E-01	1,07E-01	7	2,22E-01	7,60E-02	6
F16	30	4,45E-01	1,88E-01	4	1,56E+00	1,61E-01	7	1,67E+00	1,72E-01	8	9,54E-01	2,06E-01	6	4,74E-01	9,65E-02	5	**1,57E-32**	3,80E-33	1	2,03E-03	1,50E-03	2	2,71E-01	3,99E-02	3
F17	30	2,82E+00	4,99E-01	6	2,97E+00	2,99E-01	8	2,90E+00	2,90E-01	7	1,79E+00	9,23E-01	5	1,29E-03	3,73E-04	2	**2,15E-04**	3,73E-04	1	1,03E-01	5,51E-02	4	2,98E-03	8,95E-04	3
F18	30	**5,53E-31**	1,65E-30	1	2,03E-02	3,76E-03	6	1,62E-02	7,50E-03	5	7,53E-01	1,11E-01	8	8,67E-15	9,23E-16	3	2,09E-05	3,50E-05	4	2,57E-01	3,91E-02	7	1,20E-17	1,16E-17	2
F19	30	7,23E-01	6,77E-01	5	2,97E+01	2,82E+00	8	2,12E+01	2,02E+00	7	9,30E-01	1,49E-01	6	5,45E-01	2,06E-01	4	**1,35E-31**	3,02E-32	1	2,15E-03	1,45E-03	2	4,31E-02	1,35E-02	3
F20	30	1,20E+02	4,90E-01	8	4,70E+01	4,35E+00	6	4,43E+01	4,09E+00	5	9,80E+00	9,10E-01	4	5,22E-01	9,33E-02	3	**0,00E+00**	0,00E+00	1	4,83E-01	7,01E-02	2	5,15E+01	6,60E+00	7
F21	30	**1,15E-16**	4,82E-17	1	2,03E-07	1,83E-08	2	3,30E-06	2,97E-07	3	4,99E-02	5,28E-03	6	1,36E-01	1,28E-02	7	3,67E-02	7,86E-03	5	1,56E-01	1,48E-02	8	9,72E-03	1,21E-03	4
		Mean Rank		**3,29**			4,95			5,76			5,67			4,05			3,81			4,33			3,62
		Final Rank		**1**			6			8			7			4			3			5			2

The flow chart of the proposed pFOA versions are given in Fig. 3.

V. EXPERIMENTS

The performance of the proposed pFOA_v1 and pFOA_v2 are investigated on well-known 21 numeric benchmark function taken from [60, 61]. The benchmark functions are given in Table 1.

In this table, D denotes the dimensionality of the functions. The lower dimensional functions can be solved easily, because when we increase the dimensions of the functions, the solution space grows exponentially. Therefore, solving higher dimensional functions is harder than the solving lower dimensional functions. SR stands for search range and this means that the decision variables can take the values in this range. If a decision value is higher than upper bound or lower than lower bound, this is clamped to these values, respectively. Another important feature of the functions is the characteristics of the function and it is shown in C column. U, S, N, M mean unimodal, separable, nonseparable and multimodal. While unimodal functions have only one local minimum and it is global optimum, multimodal functions have more than one local optimum and one or more global minimum can be in these functions. Separable functions can be written as sum of a decision variables and nonseparable functions cannot be written in this form. Briefly, the performance of the proposed algorithm is investigated on these functions by using these characteristics. The benchmark problems dealt with the study are the difficult problems in terms of two different perspectives. One of them is the characteristics of the problems, unimodal and multimodal. The convergence characteristics of the compared algorithms are tested on unimodal problems, and multimodal problems examines the global search

TABLE 10. Comparative results of pFOA_v1 with other algorithms for 50 dimensions.

		pFOA_v1			SFOA			FOA			SPSO2011			FA			TSA			CS			JAYA		
Func.	D	Mean	Std. Dev.	R	Mean	Std. Dev.	R	Mean	Std. Dev.	R	Mean	Std. Dev.	R	Mean	Std. Dev.	R	Mean	Std. Dev.	R	Mean	Std. Dev.	R	Mean	Std. Dev.	R
F1	50	6,63E-116	3,07E-115	4	1,29E-07	6,51E-08	7	3,33E-06	1,57E-06	8	7,92E-295	0,00E+00	2	**0,00E+00**	0,00E+00	1	9,06E-25	8,22E-25	6	1,55E-90	2,25E-90	5	4,79E-116	1,36E-115	3
F2	50	**1,82E-111**	7,21E-111	1	8,15E-03	2,70E-03	6	7,81E-02	2,46E-02	7	8,55E+04	2,75E+04	8	1,41E-28	2,90E-28	4	9,40E-23	9,03E-23	5	5,60E-87	5,90E-87	3	3,62E-110	6,13E-110	2
F3	50	2,18E-115	6,28E-115	2	3,24E-06	8,35E-07	7	7,87E-05	1,96E-05	8	2,69E-21	1,16E-21	6	**0,00E+00**	0,00E+00	1	2,34E-25	2,35E-25	5	1,07E-90	1,00E-90	4	4,15E-114	3,92E-114	3
F4	50	**1,48E-161**	7,25E-161	1	1,08E-09	2,34E-10	3	1,58E-09	3,37E-10	4	7,81E-09	1,62E-09	5	1,84E+03	2,63E+03	7	4,56E-03	1,07E-02	6	1,00E+10	2,08E+09	8	4,38E-152	6,15E-152	2
F5	50	4,15E-51	2,90E-50	2	2,55E-03	4,91E-04	6	1,31E-02	2,48E-03	7	1,02E+01	1,98E+00	8	**4,68E-219**	0,00E+00	1	1,13E-22	5,95E-23	4	5,94E-16	8,01E-16	5	3,43E-50	6,30E-50	3
F6	50	1,43E-02	1,86E-03	3	**5,47E-05**	9,52E-06	1	4,46E-03	7,66E-04	2	1,66E+01	2,80E+00	5	4,41E+01	7,35E+00	8	2,33E+01	8,76E+00	7	2,15E+01	3,69E+00	6	5,25E-01	1,21E-01	4
F7	50	**0,00E+00**	0,00E+00	1	**0,00E+00**	0,00E+00	1	**0,00E+00**	0,00E+00	1	2,85E+01	4,82E+00	8	2,16E+00	4,35E-01	7	**0,00E+00**	0,00E+00	1	**0,00E+00**	0,00E+00	1	1,41E+00	3,81E-01	6
F8	50	3,28E-128	2,12E-127	4	8,28E-15	1,23E-15	7	5,23E-12	7,71E-13	8	4,90E+324	0,00E+00	2	**0,00E+00**	0,00E+00	1	6,83E-18	4,92E-18	6	4,37E-89	2,58E-89	5	4,69E-130	6,35E-130	3
F9	50	1,96E-01	1,13E+00	8	**2,30E-04**	3,25E-05	1	7,82E-03	1,09E-03	3	3,37E-03	4,78E-04	2	9,92E-03	1,54E-03	4	4,26E-02	1,38E-02	6	4,63E-02	6,68E-03	7	3,05E-02	6,54E-03	5
F10	50	5,79E-01	9,82E-01	2	4,86E+01	6,43E+00	8	4,80E+01	6,31E+00	6	3,52E+01	4,64E+00	5	3,31E+01	5,69E+00	4	4,81E+01	1,57E+01	7	2,88E+00	6,24E-01	3	**1,01E-01**	5,21E-02	1
F11	50	8,23E+00	4,02E+00	2	**2,59E-05**	3,26E-06	1	1,04E+02	1,42E+01	7	6,19E+01	7,81E+00	6	1,68E+02	2,10E+01	8	4,38E+01	1,35E+01	5	1,65E+01	2,22E+00	3	3,48E+01	6,11E+00	4
F12	50	1,31E+01	1,01E+01	3	**2,59E-05**	3,11E-06	1	7,10E+01	1,07E+01	5	1,03E+02	1,30E+01	7	2,04E+02	2,44E+01	8	9,65E+01	2,94E+01	6	1,19E+01	1,48E+00	2	5,29E+01	9,99E+00	4
F13	50	8,51E-16	7,04E-16	2	5,03E-09	5,81E-10	4	9,30E-08	1,07E-08	5	6,91E-03	1,29E-03	8	3,14E-03	7,26E-04	6	**0,00E+00**	0,00E+00	1	3,38E-03	9,31E-04	7	1,08E-15	3,41E-16	3
F14	50	1,98E+04	3,24E+03	6	2,09E+04	2,33E+03	8	2,04E+04	2,25E+03	7	9,00E+03	9,89E+02	5	7,67E+03	8,37E+02	4	3,77E+03	9,88E+02	2	**1,20E+03**	1,42E+02	1	4,74E+03	9,15E+02	3
F15	50	**1,90E-13**	1,86E-13	1	2,04E-04	2,19E-05	3	1,05E-03	1,12E-04	4	3,37E+00	3,63E-01	8	6,91E-02	3,68E-02	5	1,93E-13	5,45E-14	2	2,27E+00	2,51E-01	7	7,10E-01	1,38E-01	6
F16	50	6,26E-01	1,05E-01	3	1,38E+00	1,43E-01	5	1,47E+00	1,52E-01	6	5,45E+00	7,83E-01	8	1,84E+00	2,52E-01	7	1,54E-01	2,52E-01	2	**3,78E-02**	9,88E-03	1	6,35E-01	9,26E-02	4
F17	50	4,45E+00	1,18E+00	5	4,95E+00	4,98E-01	7	4,85E+00	4,85E-01	6	6,79E+01	7,58E+00	8	2,25E-01	7,28E-02	3	**7,84E-08**	5,46E-08	1	3,45E-01	1,06E-01	4	9,20E-03	2,16E-03	2
F18	50	**1,23E-52**	5,41E-52	1	2,58E-04	2,52E-05	4	1,19E-02	7,37E-03	6	6,01E+00	6,55E-01	8	4,27E-05	2,93E-05	3	1,31E-03	2,16E-03	5	1,47E-01	4,36E-02	7	2,83E-17	1,97E-17	2
F19	50	9,50E-01	6,87E-01	4	4,95E+01	4,70E+00	8	3,77E+01	3,57E+00	7	1,96E+00	2,45E-01	5	2,34E+00	4,22E-01	6	**2,90E-26**	1,25E-26	1	1,17E-02	6,60E-03	2	1,50E-01	3,72E-02	3
F20	50	2,00E+02	1,83E-01	8	8,13E+01	7,52E+00	6	8,02E+01	7,38E+00	5	2,44E+01	2,24E+00	4	4,18E+00	4,63E-01	3	**5,57E-16**	6,29E-16	1	2,75E+00	2,89E-01	2	8,90E+01	1,14E+01	7
F21	50	**3,13E-16**	1,56E-16	1	1,30E-07	1,17E-08	2	1,94E-04	1,23E-04	3	1,07E-01	9,94E-03	6	3,90E-01	3,50E-02	8	8,02E-02	1,72E-02	5	3,27E-01	2,98E-02	7	1,08E-02	1,51E-03	4
		Mean Rank		**3,05**			4,57			5,48			5,90			4,71			4,00			4,29			3,52
		Final Rank		**1**			5			7			8			6			3			4			2

capability of the algorithms. While some of the problems are unimodal, the others are multimodal. The other difficulty for the algorithms is dimensionality of the optimization problems. In our study, 10-, 30- and 50-dimensional problems are considered for verifying the search capability of the proposed algorithm. Therefore, 21 functions with different characteristics and dimensionalities are enough to test the effectiveness of the algorithms. The results obtained by pFOA_v1 and pFOA_v2 are compared with the results of the basic FOA, SFOA [62], SPSO2011 [63], FA, TSA, CS and JAYA. The reasons of the usage of these algorithms are that they are recently proposed state-of-art algorithms, in the same category with the proposed algorithm and successful in solving the problems dealt with study. The control parameters of the algorithm is population size and it is taken as 20 for all the experiments. In order to perform a fair comparison, each algorithm is run 51 times with random seeds on each function. The functions are 10, 30 and 50-dimensional functions. The maximum number of iterations is used for termination condition for the algorithms and it is taken as 100000 for 10-dimensional, 300000 for 30-dimensional, and 500000 for 50-dimensional functions.

Firstly, pFOA_v1 and pFOA_v2 are compared with the basic FOA on the 10, 30 and 50 dimensional functions in Table 2 and Table 3.

As seen from Table 2 and Table 3, the pFOA_v1 and pFOA_v2 are better than the basic version of FOA in almost all cases.

Moreover, a performance comparison between pFOA_v1 and pFOA_v2 has been performed in order to show the better approach. The comparison results of the versions are reported in Table 4 on the test functions.

As seen from Table 4, pFOA_v2 produces better results than pFOA_v1 in almost all cases.

Before the proposed method is compared with the state-of-art algorithms, the proposed algorithm is compared with the JAYA and basic FOA on the 10, 30 and 50 dimensional functions in Table 5, Table 6 and Table 7, respectively.

In accordance with Table 5, 6, and 7, the proposed algorithm is better than the JAYA and FOA because the proposed algorithm uses the effective approach in the FOA and JAYA.

Another important comparison metric in this type of algorithms is convergence characteristics. The convergence comparison of pFOA_v1 and basic algorithm are given in Fig. 4 for 10-dimensional, Fig. 5 for 30-dimensional and Fig. 6 for 50-dimensional functions.

Based in Fig. 4, 5 and 6, the convergence characteristics of the proposed algorithm is good at achieving to optimum or near optimum solution. On almost all cases, the pFOA_v1 is better than the basic version of the algorithm in terms of this measurement.

In the third comparison, the pFOA_v1 is compared with the state-of-art algorithms in Table 8 for 10-dimension functions, Table 9 for 30-dimensional functions and Table 10 for 50-dimensional functions.

When we consider the comparison tables, the rank of pFOA_v1 is 1, and therefore, the mean rank value of pFOA_v1 is better than other algorithms. When the dimensionalities of the functions are increased, the performance of the algorithms are decreased but the pFOA_v1 shows better performance than the compared algorithms.

In the fourth comparison, the pFOA_v2 is compared with same algorithms in Table 11 for 10-dimensional functions, in Table 12 for 30-dimensional functions and in Table 13 for 50-dimensional functions.

As we see the comparison of the two versions of the proposed algorithms, the second version is better than the first version, and therefore, the mean rank of the second version is better than both first version and compared algorithms.

Moreover, the execution times of the compared algorithms on dimensions 10, 30 and 50 are given in Fig. 7. In Fig. 7, the reported times are mean running times of 21 benchmark

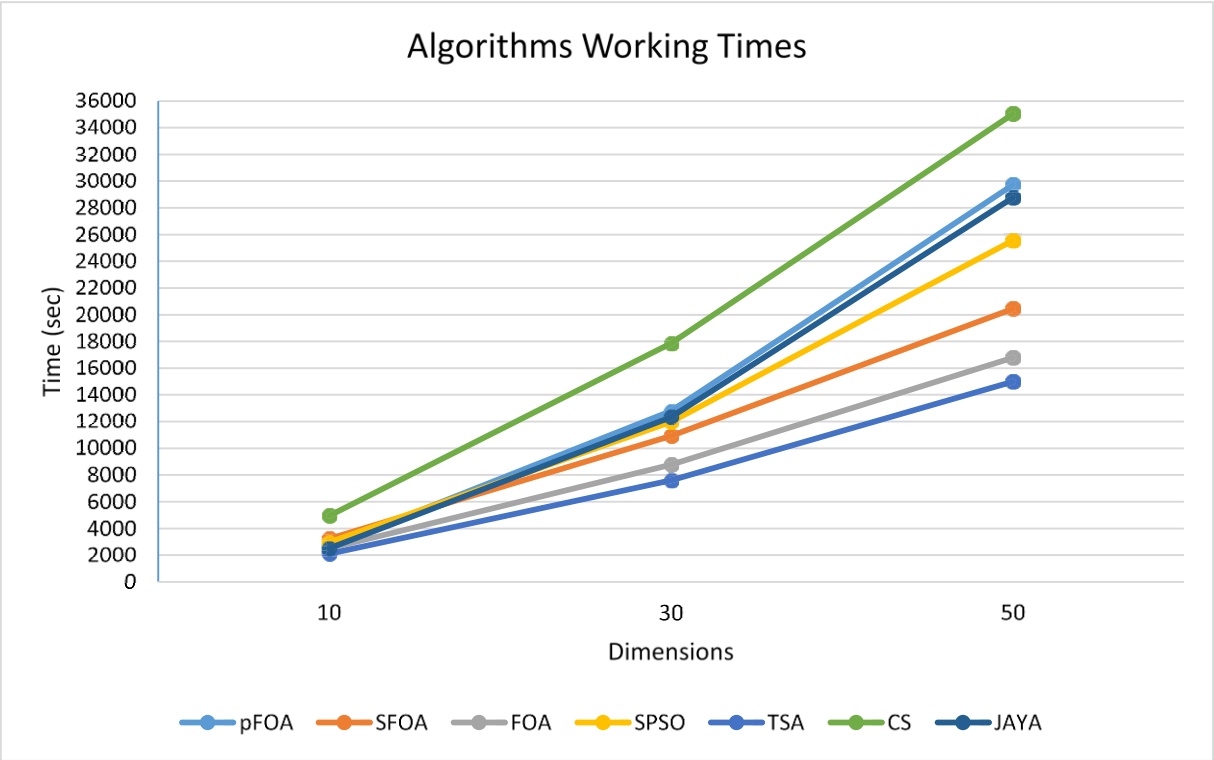

FIGURE 7. The working times of the algorithms.

TABLE 11. Comparative results of pFOA_v2 with other algorithms for 10 dimensions.

		pFOA_v2			SFOA			FOA			SPSO2011			FA			TSA			CS			JAYA		
Func.	D	Mean	Std. Dev.	R	Mean	Std. Dev.	R	Mean	Std. Dev.	R	Mean	Std. Dev.	R	Mean	Std. Dev.	R	Mean	Std. Dev.	R	Mean	Std. Dev.	R	Mean	Std. Dev.	R
F1	10	2,32E-158	1,62E-157	2	5,05E-07	2,54E-07	7	3,21E-06	1,52E-06	8	**2,50E-280**	0,00E+00	1	7,01E-88	2,70E-88	6	2,89E-109	8,34E-109	4	1,09E-90	2,49E-90	5	1,99E-151	7,85E-151	3
F2	10	**2,09E-155**	1,05E-154	1	4,19E-02	1,39E-02	6	1,18E-01	3,72E-02	7	9,98E+03	3,66E+03	8	1,89E-83	5,81E-84	5	1,24E-106	3,23E-106	3	5,81E-88	4,72E-88	4	5,10E-148	6,63E-148	2
F3	10	6,89E-156	2,69E-155	2	2,75E-06	7,09E-07	7	1,74E-05	4,35E-06	8	**3,21E-188**	0,00E+00	1	3,43E-89	8,16E-90	6	1,15E-110	2,59E-110	4	5,59E-92	5,85E-92	5	6,44E-152	5,21E-152	3
F4	10	**2,09E-271**	0,00E+00	1	2,68E-08	5,85E-09	7	4,06E-08	8,66E-09	8	5,22E-09	1,17E-09	6	6,31E-23	9,19E-23	5	4,07E-169	0,00E+00	3	8,20E-140	1,25E-139	4	8,63E-267	0,00E+00	2
F5	10	7,59E-78	8,91E-78	2	2,28E-03	4,38E-04	7	5,87E-03	1,11E-03	8	1,93E-71	2,55E-71	3	6,61E-45	1,20E-45	5	3,40E-69	2,35E-69	4	7,53E-42	2,15E-42	6	**3,50E-79**	2,69E-79	1
F6	10	2,01E-47	3,48E-47	2	2,44E-04	4,25E-05	7	1,12E-03	1,92E-04	8	**1,27E-116**	1,45E-116	1	1,51E-44	2,49E-45	4	9,54E-20	8,23E-20	6	3,17E-26	1,60E-26	5	6,28E-45	3,77E-45	3
F7	10	**0,00E+00**	0,00E+00	1	**0,00E+00**	0,00E+00	1	**0,00E+00**	0,00E+00	1	**0,00E+00**	0,00E+00	1	1,57E-01	6,82E-02	8	**0,00E+00**	0,00E+00	1	**0,00E+00**	0,00E+00	1	**0,00E+00**	0,00E+00	1
F8	10	2,38E-265	0,00E+00	2	1,40E-13	2,08E-14	7	5,93E-12	8,75E-13	8	**0,00E+00**	0,00E+00	1	1,63E-182	0,00E+00	4	1,66E-161	2,41E-161	6	8,92E-170	0,00E+00	5	4,46E-249	0,00E+00	3
F9	10	8,08E-04	4,53E-04	4	2,52E-04	3,67E-05	2	2,77E-03	3,93E-04	8	2,55E-04	3,94E-05	3	**1,31E-04**	2,13E-05	1	1,03E-03	3,49E-04	6	1,01E-03	1,55E-04	5	1,08E-03	2,18E-04	7
F10	10	5,69E+00	2,92E-01	6	8,90E+00	1,18E+00	8	6,99E+00	9,19E-01	7	2,56E-01	1,27E-01	3	1,05E+00	2,55E-01	4	1,83E+00	5,96E-01	5	**2,55E-19**	2,39E-19	1	1,10E-03	1,05E-03	2
F11	10	6,15E-01	6,45E-01	3	1,03E-04	1,30E-05	2	5,58E+00	1,04E+00	6	8,92E+00	1,40E+00	7	1,20E+01	1,64E+00	8	1,81E+00	7,33E-01	4	**0,00E+00**	0,00E+00	1	3,81E+00	7,77E-01	5
F12	10	3,49E-01	2,61E-01	2	6,55E+00	1,89E+00	6	1,38E+00	2,09E-01	3	7,10E+00	8,59E-01	7	1,20E+01	1,56E+00	8	5,97E+00	1,71E+00	5	**2,94E-01**	7,28E-02	1	5,77E+00	9,95E-01	4
F13	10	**0,00E+00**	0,00E+00	1	5,99E-08	6,92E-09	3	2,85E-07	3,28E-08	4	4,16E-02	5,46E-03	6	6,39E-02	8,14E-03	8	4,60E-02	1,81E-02	7	2,82E-03	5,94E-04	5	**0,00E+00**	0,00E+00	1
F14	10	3,96E+03	1,03E+03	5	4,19E+03	4,66E+02	8	4,00E+03	4,41E+02	6	1,04E+03	1,17E+02	4	9,06E+02	1,02E+02	3	1,16E+01	9,88E+00	2	**1,62E-11**	1,28E-11	1	4,15E+03	6,33E+02	7
F15	10	**1,67E-15**	6,20E-16	1	9,10E-04	9,76E-05	6	2,35E-03	2,50E-04	7	2,26E-02	1,71E-02	8	8,43E-15	9,72E-16	5	3,55E-15	8,87E-16	3	3,34E-15	3,65E-16	2	3,76E-15	5,68E-16	4
F16	10	9,15E-03	2,39E-03	4	2,48E+00	2,57E-01	7	2,65E+00	2,74E-01	8	6,10E-03	4,47E-03	3	1,83E-02	7,70E-03	5	**4,71E-32**	1,14E-32	1	**4,71E-32**	4,83E-33	1	2,07E-02	3,11E-03	6
F17	10	9,67E-01	2,29E-01	7	9,89E-01	9,94E-02	8	9,54E-01	9,57E-02	6	6,46E-04	2,65E-04	5	**1,35E-32**	1,33E-33	1	**1,35E-32**	3,18E-33	1	**1,35E-32**	1,34E-33	1	2,15E-04	2,15E-04	4
F18	10	1,26E-03	5,10E-04	4	1,33E-02	1,50E-03	7	3,00E-03	1,69E-03	5	5,49E-03	1,67E-03	6	**5,13E-16**	8,57E-17	1	7,93E-13	1,33E-12	2	2,36E-02	3,59E-03	8	5,29E-05	4,51E-05	3
F19	10	1,00E+00	2,68E-01	6	1,00E+01	9,53E-01	8	5,98E+00	5,82E-01	7	3,80E-02	1,99E-02	5	2,15E-03	1,44E-03	3	**1,35E-31**	3,02E-32	1	**1,35E-31**	1,27E-32	1	2,15E-03	2,04E-03	3
F20	10	**0,00E+00**	0,00E+00	1	1,39E+01	1,29E+00	8	1,02E+01	9,38E-01	6	3,48E-01	5,22E-02	5	4,88E-16	1,36E-16	4	**0,00E+00**	0,00E+00	1	**0,00E+00**	0,00E+00	1	1,04E+01	1,61E+00	7
F21	10	**0,00E+00**	0,00E+00	1	5,05E-07	4,56E-08	2	3,21E-06	2,89E-07	3	1,03E-02	9,75E-04	7	1,03E-02	9,71E-04	8	9,72E-03	2,07E-03	5	9,72E-03	8,69E-04	6	9,72E-03	1,21E-03	4
		Mean Rank		**2,76**			5,90			6,29			4,33			4,86			3,52			3,29			3,57
		Final Rank		**1**			7			8			5			6			3			2			4

functions. The applications were run in a computer with Intel Coffee Lake Core i7-8700K, 6GB GDDR5 Nvidia GTX1060 O.C 192 Bit, 17.3" FHD 1920×1080 G-SYNC IPS Mat LED Display, 16GB (2×8GB) DDR4 2666MHz, 512GB SAMSUNG PM981 M.2 SSD PCIe 3.0 × 4 .

Time measurement is as follows. For each algorithm; firstly the initialization time is calculated for 30 times, and the mean of these initializations times is obtained. The time of one iteration is obtained for 30 times, and then this time is extended for 51 iteration. The mean of running time of 30 runs is calculated. The reported solutions are the sum of mean of initialization time and mean of running time.

The running time of FA is higher than all compared algorithm and therefore, it is not included in the time graphic

TABLE 12. Comparative results of pFOA_v2 with other algorithms for 30 dimensions.

		pFOA_v2			SFOA			FOA			SPSO2011			FA			TSA			CS			JAYA		
Func.	D	Mean	Std. Dev.	R	Mean	Std. Dev.	R	Mean	Std. Dev.	R	Mean	Std. Dev.	R	Mean	Std. Dev.	R	Mean	Std. Dev.	R	Mean	Std. Dev.	R	Mean	Std. Dev.	R
F1	30	4,53E-197	0,00E+00	3	2,03E-07	1,02E-07	7	3,30E-06	1,56E-06	8	**1,29E-303**	0,00E+00	1	4,99E-262	0,00E+00	2	1,65E-56	2,72E-56	6	3,79E-94	1,12E-93	5	9,97E-146	2,13E-145	4
F2	30	2,97E-198	0,00E+00	2	1,31E-02	4,34E-03	6	8,36E-02	2,63E-02	7	3,96E+04	1,28E+04	8	**8,13E-257**	0,00E+00	1	2,99E-54	3,64E-54	5	8,64E-92	6,38E-92	4	5,83E-141	1,49E-140	3
F3	30	2,84E-179	0,00E+00	2	3,09E-06	7,96E-07	7	4,80E-05	1,20E-05	8	4,30E-41	2,04E-41	6	**6,92E-263**	0,00E+00	1	1,81E-57	2,00E-57	5	3,09E-96	1,89E-96	4	2,56E-144	3,51E-144	3
F4	30	**3,02E-257**	0,00E+00	1	2,99E-09	6,51E-10	5	4,43E-09	9,43E-10	6	1,02E-08	2,16E-09	7	1,12E-16	1,02E-16	4	4,70E-42	1,20E-41	3	1,00E+10	2,08E+09	8	1,69E-223	0,00E+00	2
F5	30	1,39E-97	9,91E-97	2	2,48E-03	4,77E-04	6	1,02E-02	1,92E-03	7	3,29E+00	6,95E-01	8	**9,38E-132**	1,69E-132	1	1,14E-42	6,51E-43	5	3,02E-52	8,49E-53	4	1,08E-81	1,15E-81	3
F6	30	**1,38E-09**	3,16E-09	1	8,92E-05	1,55E-05	2	2,44E-03	4,18E-04	4	7,25E+00	1,31E+00	6	1,25E+01	2,18E+00	8	6,28E-01	2,51E-01	5	8,78E+00	1,61E+00	7	1,81E-04	6,62E-05	3
F7	30	**0,00E+00**	0,00E+00	1	**0,00E+00**	0,00E+00	1	**0,00E+00**	0,00E+00	1	4,84E+00	9,77E-01	8	2,94E-01	9,88E-02	7	**0,00E+00**	0,00E+00	1	**0,00E+00**	0,00E+00	1	1,37E-01	8,06E-02	6
F8	30	5,80E-258	0,00E+00	3	2,07E-14	3,09E-15	7	5,35E-12	7,89E-13	8	**0,00E+00**	0,00E+00	1	**0,00E+00**	0,00E+00	1	4,60E-55	8,18E-55	6	1,91E-107	1,29E-107	5	4,15E-183	0,00E+00	4
F9	30	3,86E-03	1,45E-03	4	**2,35E-04**	3,36E-05	1	5,78E-03	8,09E-04	5	1,53E-03	2,31E-04	2	2,09E-03	3,77E-04	3	8,42E-03	2,75E-03	6	1,17E-02	1,78E-03	8	9,21E-03	2,28E-03	7
F10	30	2,67E+01	2,95E-01	6	2,87E+01	3,81E+00	8	2,75E+01	3,61E+00	7	1,84E+01	2,95E+00	4	1,76E+01	3,47E+00	3	2,15E+01	6,88E+00	5	6,25E-01	2,07E-01	2	**1,17E-09**	8,47E-10	1
F11	30	2,46E+00	2,50E+00	2	**4,08E-05**	5,14E-06	1	4,16E+01	6,21E+00	7	3,35E+01	4,29E+00	6	7,54E+01	9,61E+00	8	2,68E+01	1,09E+01	5	4,19E+00	6,48E-01	3	2,03E+01	3,61E+00	4
F12	30	1,30E+01	5,37E+00	3	**4,08E-05**	4,91E-06	1	1,49E+01	3,32E+00	4	4,59E+01	5,73E+00	6	1,04E+02	1,26E+01	8	6,03E+01	1,85E+01	7	5,06E+00	6,47E-01	2	2,94E+01	5,46E+00	5
F13	30	**0,00E+00**	0,00E+00	1	1,16E-08	1,34E-09	3	1,38E-07	1,58E-08	4	9,75E-03	1,56E-03	8	4,83E-03	9,02E-04	7	1,45E-04	2,87E-04	5	2,13E-03	6,74E-04	6	4,79E-17	1,61E-17	2
F14	30	8,96E+03	2,34E+03	5	1,26E+04	1,40E+03	8	1,22E+04	1,35E+03	7	4,89E+03	5,41E+02	4	4,16E+03	4,58E+02	3	1,30E+03	3,48E+02	2	**3,13E+02**	4,27E+01	1	1,05E+04	1,72E+03	6
F15	30	**3,62E-15**	9,13E-16	1	3,31E-04	3,55E-05	4	1,35E-03	1,44E-04	5	2,31E+00	2,53E-01	8	2,48E-14	2,78E-15	3	6,41E-15	1,64E-15	2	7,41E-01	1,07E-01	7	2,22E-01	7,60E-02	6
F16	30	7,56E-02	1,88E-02	3	1,56E+00	1,61E-01	7	1,67E+00	1,72E-01	8	9,54E-01	2,06E-01	6	4,74E-01	9,65E-02	5	**1,57E-32**	3,80E-33	1	2,03E-03	1,50E-03	2	2,71E-01	3,99E-02	4
F17	30	2,81E+00	6,64E-01	6	2,97E+00	2,99E-01	8	2,90E+00	2,90E-01	7	1,79E+00	9,23E-01	5	1,29E-03	3,73E-04	2	**2,15E-04**	3,73E-04	1	1,03E-01	5,51E-02	4	2,98E-03	8,95E-04	3
F18	30	3,43E-04	1,98E-04	4	2,03E-02	3,76E-03	6	1,62E-02	7,50E-03	5	7,53E-01	1,11E-01	8	8,67E-15	9,23E-16	2	2,09E-05	3,50E-05	3	2,57E-01	3,91E-02	7	**1,20E-17**	1,16E-17	1
F19	30	8,43E+00	1,92E+00	6	2,97E+01	2,82E+00	8	2,12E+01	2,02E+00	7	9,30E-01	1,49E-01	5	5,45E-01	2,06E-01	4	**1,35E-31**	3,02E-32	1	2,15E-03	1,45E-03	2	4,31E-02	1,35E-02	3
F20	30	**0,00E+00**	0,00E+00	1	4,70E+01	4,35E+00	7	4,43E+01	4,09E+00	6	9,80E+00	9,10E-01	5	5,22E-01	9,33E-02	4	**0,00E+00**	0,00E+00	1	4,83E-01	7,01E-02	3	5,15E+01	6,60E+00	8
F21	30	**0,00E+00**	0,00E+00	1	2,03E-07	1,83E-08	2	3,30E-06	2,97E-07	3	4,99E-02	5,28E-03	6	1,36E-01	1,28E-02	7	3,67E-02	7,86E-03	5	1,56E-01	1,48E-02	8	9,72E-03	1,21E-03	4
		Mean Rank		**2,76**			5,00			5,90			5,62			4,00			3,81			4,43			3,90
		Final Rank		**1**			6			8			7			4			2			5			3

TABLE 13. Comparative results of pFOA_v2 with other algorithms for 50 dimensions.

		pFOA_v2			SFOA			FOA			SPSO2011			FA			TSA			CS			JAYA		
Func.	D	Mean	Std. Dev.	R	Mean	Std. Dev.	R	Mean	Std. Dev.	R	Mean	Std. Dev.	R	Mean	Std. Dev.	R	Mean	Std. Dev.	R	Mean	Std. Dev.	R	Mean	Std. Dev.	R
F1	50	1,97E-164	0,00E+00	3	1,29E-07	6,51E-08	7	3,33E-06	1,57E-06	8	7,92E-295	0,00E+00	2	**0,00E+00**	0,00E+00	1	9,06E-25	8,22E-25	6	1,55E-90	2,25E-90	5	4,79E-116	1,36E-115	4
F2	50	**3,50E-228**	0,00E+00	1	8,15E-03	2,70E-03	6	7,81E-02	2,46E-02	7	8,55E+04	2,75E+04	8	1,41E-28	2,90E-28	4	9,40E-23	9,03E-23	5	5,60E-87	5,90E-87	3	3,62E-110	6,13E-110	2
F3	50	1,56E-163	0,00E+00	2	3,24E-06	8,35E-07	7	7,87E-05	1,96E-05	8	2,69E-21	1,16E-21	6	**0,00E+00**	0,00E+00	1	2,34E-25	2,35E-25	5	1,07E-90	1,00E-90	4	4,15E-114	3,92E-114	3
F4	50	**6,88E-236**	0,00E+00	1	1,08E-09	2,34E-10	3	1,58E-09	3,37E-10	4	7,81E-09	1,62E-09	5	1,84E+03	2,63E+03	7	4,56E-03	1,07E-02	6	1,00E+10	2,08E+09	8	4,38E-152	6,15E-152	2
F5	50	7,93E-137	5,66E-136	2	2,55E-03	4,91E-04	6	1,31E-02	2,48E-03	7	1,02E+01	1,98E+00	8	**4,68E-219**	0,00E+00	1	1,13E-22	5,95E-23	4	5,94E-16	8,01E-16	5	3,43E-50	6,30E-50	3
F6	50	**2,56E-07**	3,94E-07	1	5,47E-05	9,52E-06	2	4,46E-03	7,66E-04	3	1,66E+01	2,80E+00	5	4,41E+01	7,35E+00	8	2,33E+01	8,76E+00	7	2,15E+01	3,69E+00	6	5,25E-01	1,21E-01	4
F7	50	**0,00E+00**	0,00E+00	1	**0,00E+00**	0,00E+00	1	**0,00E+00**	0,00E+00	1	2,85E+01	4,82E+00	8	2,16E+00	4,35E-01	7	**0,00E+00**	0,00E+00	1	**0,00E+00**	0,00E+00	1	1,41E+00	3,81E-01	6
F8	50	7,75E-180	0,00E+00	3	8,28E-15	1,23E-15	7	5,23E-12	7,71E-13	8	4,90E-324	0,00E+00	2	**0,00E+00**	0,00E+00	1	6,83E-18	4,92E-18	6	4,37E-89	2,58E-89	5	4,69E-130	6,35E-130	4
F9	50	5,05E-03	3,22E-03	3	**2,30E-04**	3,25E-05	1	7,82E-03	1,09E-03	4	3,37E-03	4,78E-04	2	9,92E-03	1,54E-03	5	4,26E-02	1,38E-02	7	4,63E-02	6,68E-03	8	3,05E-02	6,54E-03	6
F10	50	4,69E+01	1,41E+01	5	4,86E+01	6,43E+00	8	4,80E+01	6,31E+00	6	3,52E+01	4,64E+00	4	3,31E+01	5,69E+00	3	4,81E+01	1,57E+01	7	2,88E+00	6,24E-01	2	**1,01E-01**	5,21E-02	1
F11	50	4,34E+00	3,61E+00	2	**2,59E-05**	3,26E-06	1	1,04E+02	1,42E+01	7	6,19E+01	7,81E+00	6	1,68E+02	2,10E+01	8	4,38E+01	1,35E+01	5	1,65E+01	2,22E+00	3	3,48E+01	6,11E+00	4
F12	50	2,40E+01	9,76E+00	3	**2,59E-05**	3,11E-06	1	7,10E+01	1,07E+01	5	1,03E+02	1,30E+01	7	2,04E+02	2,44E+01	8	9,65E+01	2,94E+01	6	1,19E+01	1,48E+00	2	5,29E+01	9,99E+00	4
F13	50	**0,00E+00**	0,00E+00	1	5,03E-09	5,81E-10	4	9,30E-08	1,07E-08	5	6,91E-03	1,29E-03	8	3,14E-03	7,26E-04	6	**0,00E+00**	0,00E+00	1	3,38E-03	9,31E-04	7	1,08E-15	3,41E-16	3
F14	50	1,55E+04	4,00E+03	6	2,09E+04	2,33E+03	8	2,04E+04	2,25E+03	7	9,00E+03	9,89E+02	5	7,67E+03	8,37E+02	4	3,77E+03	9,88E+02	2	**1,20E+03**	1,42E+02	1	4,74E+03	9,15E+02	3
F15	50	**4,67E-15**	1,24E-15	1	2,04E-04	2,19E-05	3	1,05E-03	1,12E-04	4	3,37E+00	3,63E-01	8	6,91E-02	3,68E-02	5	1,93E-13	5,45E-14	2	2,27E+00	2,51E-01	7	7,10E-01	1,38E-01	6
F16	50	1,52E-01	2,29E-02	2	1,38E+00	1,43E-01	5	1,47E+00	1,52E-01	6	5,45E+00	7,83E-01	8	1,84E+00	2,52E-01	7	1,54E-01	2,52E-01	3	**3,78E-02**	9,88E-03	1	6,35E-01	9,26E-02	4
F17	50	4,73E+00	1,88E-01	5	4,95E+00	4,98E-01	7	4,85E+00	4,85E-01	6	6,79E+01	7,58E+00	8	2,25E-01	7,28E-02	3	**7,84E-08**	5,46E-08	1	3,45E-01	1,06E-01	4	9,20E-03	2,16E-03	2
F18	50	2,25E-05	7,15E-05	2	2,58E-04	2,52E-05	4	1,19E-02	7,37E-03	6	6,01E+00	6,55E-01	8	4,27E-05	2,93E-05	3	1,31E-03	2,16E-03	5	1,47E-01	4,36E-02	7	**2,83E-17**	1,97E-17	1
F19	50	2,40E+01	5,39E+00	6	4,95E+01	4,70E+00	8	3,77E+01	3,57E+00	7	1,96E+00	2,45E-01	4	2,34E+00	4,22E-01	5	**2,90E-26**	1,25E-26	1	1,17E-02	6,60E-03	2	1,50E-01	3,72E-02	3
F20	50	**0,00E+00**	0,00E+00	1	8,13E+01	7,52E+00	6	8,02E+01	7,38E+00	5	2,44E+01	2,24E+00	4	4,18E+00	4,63E-01	3	5,57E-16	6,29E-16	2	2,75E+00	2,89E-01	2	8,90E+01	1,14E+01	7
F21	50	**1,20E-17**	5,60E-18	1	1,30E-07	1,17E-08	2	1,94E-04	1,23E-04	3	1,07E-01	9,94E-03	6	3,90E-01	3,50E-02	8	8,02E-02	1,72E-02	5	3,27E-01	2,98E-02	7	1,08E-02	1,51E-03	4
		Mean Rank		**2,48**			4,62			5,57			5,81			4,67			4,14			4,29			3,62
		Final Rank		**1**			5			7			8			6			3			4			2

to prevent scaling issues. However, the running times of FA are 26469 seconds on 10-dimensional, 61200 seconds on 30-dimensional, and 79081 seconds on 50-dimensional 21 problems.

VI. RESULTS AND DISCUSSION

The proposed pFOA versions have been compared with the basic FOA, modified FOA called SFOA and other intelligent optimization algorithms including SPSO2011, FA, TSA, CS and JAYA to validate the effectiveness of pFOA versions. The comparison results are shown in Table 2-13. The value highlighted in bold indicates the algorithm with the better value in the benchmark function. pFOAs are more effective and efficient in solving the benchmark functions than the basic FOA. This situation is clearly seen in Table 2 and Table 3. pFOA_v1 has achieved the best values in all dimensionalities in almost all cases, with the exception of 3 functions. pFOA_v2 has achieved the best values in all dimensionalities in almost all cases. Hence, pFOAs have a higher accuracy of optimization than the basic FOA. By looking at the convergence graphs, it appears that pFOAs converges to optimum or near optimum solutions faster than the basic FOA. As a result, the performance of pFOA versions on benchmark functions are better than the basic FOA and compared 6 algorithms in terms of solution quality and robustness. When we discuss all the results, it should be mentioned that the better or comparable results are obtained by pFOA versions. The reason of this result is that the solution update rule is changed with best-worst-based solution strategy, and the agents consider not only best solution but also

worst solution. Therefore, the solution space is effectively searched and novel solutions are easily discovered by using this rule. This improves the global search capability but decreases the fast convergence. This trade-off between global search and convergence are acceptable level in the proposed algorithms in contrast to basic version of the algorithm because the basic algorithm tends to improve local search capability and fast convergence due to following the best solutions continuously. Especially, it is seen that the proposed algorithms are more effective than the basic version in solving multimodal problems when the results and convergence graphs of functions are analyzed.

VII. CONCLUSION AND FUTURE WORKS

This study focuses on the improvement of FOA by proposing a novel solution update rule based on best and worst solutions. The proposed algorithms are called pFOA_v1 and pFOA_v2, and they are applied to solve 21 well-known numeric benchmark functions. The proposed algorithms are compared with the basic FOA and state-of-art swarm intelligence algorithms. The experimental results show that the pFOA versions produce competitive and comparable solutions for the numeric functions. The reason of these results is that the pFOA versions follow best and worst solutions in the populations and improve global search capability of the basic algorithm. Briefly, the basic FOA's update strategy has been renewed and improved by considering global search capability. At the same time, an important problem in the basic FOA (the solutions cannot be negative values) and in the novel versions this problem has been solved. The performance of pFOA versions have been compared with basic FOA, SFOA, SPSO2011, FA, TSA, CS and JAYA algorithms. According to the test results, pFOA versions have improved the convergence characteristics and global search ability of basic FOA. The performance of pFOA versions is better than other algorithms in terms of rank-based sorting and convergence characteristics. In near future, the pFOA versions will be applied to solve constrained optimization problems and modified for solving binary optimization problems due to the fact that they have a simple algorithmic structure, few control parameters, and high performance on unconstrained continuous optimization in this study.

COMPLIANCE WITH ETHICAL STANDARDS

Funding : This study is not funding.

Conflict of Interest: The authors declare that there is no conflict of interest regarding the publication of this paper.

Ethical Approval: This article does not contain any studies with human participants or animals performed by any of the authors.

Data Availability Statement: The data used to support the findings of this study are included within the article.

REFERENCES

[1] J. Kennedy and R. Eberhart, "Particle swarm optimization," in *Proc. IEEE Int. Conf. Neural Netw.*, Perth, WA, Australia, vol. 4, Nov./Dec. 1995, pp. 1942–1948.

[2] X.-S. Yang, "Firefly algorithms for multimodal optimization," in *Proc. Int. Symp. Stochastic Algorithms*. Berlin, Germany: Springer, 2009, pp. 169–178.

[3] X.-S. Yang and S. Deb, "Cuckoo search via Lévy flights," in *Proc. World Congr. Nature Biol. Inspired Comput. (NaBIC)*, Dec. 2009, pp. 210–214.

[4] M. Dorigo, V. Maniezzo, and A. Colorni, "Ant system: Optimization by a colony of cooperating agents," *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 26, no. 1, pp. 29–41, Feb. 1996.

[5] D. Karaboga, "An idea based on honey bee swarm for numerical optimization," Dept. Comput. Eng., Erciyes Univ., Kayseri, Turkey, Tech. Rep. TR06, 2005.

[6] W.-T. Pan, "A new fruit fly optimization algorithm: Taking the financial distress model as an example," *Knowl.-Based Syst.*, vol. 26, pp. 69–74, Feb. 2012.

[7] M. S. Kiran, "TSA: Tree-seed algorithm for continuous optimization," *Expert Syst. Appl.*, vol. 42, no. 19, pp. 6686–6698, 2015.

[8] W.-Y. Lin and T.-H. Huang, "A hybrid approach of 3-D fruit fly optimization algorithm and general regression neural network for financial distress forecasting," *Int. J. Digit. Content Technol. Appl.*, vol. 8, no. 4, p. 1, 2014.

[9] P.-W. Chen, W.-Y. Lin, T.-H. Huang, and W.-T. Pan, "Using fruit fly optimization algorithm optimized grey model neural network to perform satisfaction analysis for E-business service," *Appl. Math. Inf. Sci.*, vol. 7, no. 2L, pp. 459–465, 2013.

[10] T.-H. Huang, Y. Leu, and W.-T. Pan, "Constructing ZSCORE-based financial crisis warning models using fruit fly optimization algorithm and general regression neural network," *Kybernetes*, vol. 45, no. 4, pp. 650–665, 2016.

[11] H.-Z. Li, S. Guo, C.-J. Li, and J.-Q. Sun, "A hybrid annual power load forecasting model based on generalized regression neural network with fruit fly optimization algorithm," *Knowl.-Based Syst.*, vol. 37, pp. 378–387, Jan. 2013.

[12] H. Zhao, S. Guo, and W. Xue, "Urban saturated power load analysis based on a novel combined forecasting model," *Information*, vol. 6, no. 1, pp. 69–88, 2015.

[13] L. Jebaraj, C. C. A. Rajan, and I. Soubache, "Voltage and real power loss analysis incorporating CE-SSSC with VS-SVC combination through fruit fly optimization," *WSEAS Trans. Power Syst. J.*, vol. 10, no. 1, pp. 55–72, 2015.

[14] P. Zhang and L. Wang, "Grouped fruit-fly optimization algorithm for the no-wait lot streaming flow shop scheduling," in *Proc. Int. Conf. Intell. Comput.* Cham, Switzerland: Springer, 2014, pp. 664–674.

[15] A. Rana and A. Sharma, "Resolving set-streaming stream-shop scheduling in distributed system by mean of an a FOA," *Int. J. Comput. Sci. Eng. Technol.*, vol. 5, no. 4, pp. 394–403, 2014.

[16] X. Ning and H. Hu, "Multiple-population fruit fly optimization algorithm for scheduling problem of order picking operation in automatic warehouse," *J. Lanzhou Jiaotong Univ.*, vol. 3, p. 22, 2014.

[17] L. Wang and X.-L. Zheng, "A knowledge-guided multi-objective fruit fly optimization algorithm for the multi-skill resource constrained project scheduling problem," *Swarm Evol. Comput.*, vol. 38, pp. 54–63, Feb. 2018.

[18] X.-L. Zheng and L. Wang, "A collaborative multiobjective fruit fly optimization algorithm for the resource constrained unrelated parallel machine green scheduling problem," *IEEE. Trans. Syst., Man, Cybern., Syst.*, vol. 48, no. 5, pp. 790–800, May 2018.

[19] J.-Q. Li, Q.-K. Pan, and K. Mao, "A hybrid fruit fly optimization algorithm for the realistic hybrid flowshop rescheduling problem in steelmaking systems," *IEEE Trans. Automat. Sci. Eng.*, vol. 13, no. 2, pp. 932–949, Apr. 2016.

[20] Y. Xing, "Design and optimization of key control characteristics based on improved fruit fly optimization algorithm," *Kybernetes*, vol. 42, no. 3, pp. 466–481, 2013.

[21] Y. Liu, X. Wang, and Y. Li, "A modified fruit-fly optimization algorithm aided PID controller designing," in *Proc. 10th World Congr. Intell. Control Automat. (WCICA)*, Jul. 2012, pp. 233–238.

[22] W. Sheng and Y. Bao, "Fruit fly optimization algorithm based fractional order fuzzy-PID controller for electronic throttle," *Nonlinear Dyn.*, vol. 73, nos. 1–2, pp. 611–619, 2013.

[23] L. S. Wei, X. Wu, M. Q. Niu, and Z. Y. Chen, "FOA based PID controller for human balance keeping," *Appl. Mech. Mater.*, vols. 494–495, pp. 1072–1075, Feb. 2014.

[24] X. Liu, Y. Shi, and J. Xu, "Parameters tuning approach for proportion integration differentiation controller of magnetorheological fluids brake based on improved fruit fly optimization algorithm," *Symmetry*, vol. 9, no. 7, p. 109, 2017.

[25] W. Wang and X. Liu, "Melt index prediction by least squares support vector machines with an adaptive mutation fruit fly optimization algorithm," *Chemometrics Intell. Lab. Syst.*, vol. 141, pp. 79–87, Feb. 2015.
[26] X.-L. Zheng, L. Wang, and S.-Y. Wang, "A novel fruit fly optimization algorithm for the semiconductor final testing scheduling problem," *Knowl.-Based Syst.*, vol. 57, pp. 95–103, Feb. 2014.
[27] L. Wang, S.-X. Lv, and Y.-R. Zeng, "Effective sparse adaboost method with ESN and FOA for industrial electricity consumption forecasting in China," *Energy*, vol. 155, pp. 1013–1031, Jul. 2018.
[28] S.-M. Lin, "Analysis of service satisfaction in Web auction logistics service using a combination of fruit fly optimization algorithm and general regression neural network," *Neural Comput. Appl.*, vol. 22, nos. 3–4, pp. 783–791, 2013.
[29] Z. Z. Abidin, M. R. Arshad, and U. K. Ngah, "A simulation based fly optimization algorithm for swarms of mini autonomous surface vehicles application," *Indian J. Geo-Mar. Sci.*, vol. 40, no. 2, pp. 250–266, Apr. 2011.
[30] L. Si, Z. Wang, X. Liu, C. Tan, Z. Liu, and J. Xu, "Identification of shearer cutting patterns using vibration signals based on a least squares support vector machine with an improved fruit fly optimization algorithm," *Sensors*, vol. 16, no. 1, p. 90, 2016.
[31] D. Niu, Y. Li, S. Dai, H. Kang, Z. Xue, X. Jin, and Y. Song, "Sustainability evaluation of power grid construction projects using improved TOPSIS and least square support vector machine with modified fly optimization algorithm," *Sustainability*, vol. 10, no. 1, p. 231, 2018.
[32] W.-T. Pan, "Mixed modified fruit fly optimization algorithm with general regression neural network to build oil and gold prices forecasting model," *Kybernetes*, vol. 43, no. 7, pp. 1053–1063, 2014.
[33] S.-C. Yang, C.-S. Lee, and H.-S. Lee, "Evaluation of logistic flow service satisfaction using the evolutionary computation technique and general regression neural network technique," *Int. J. Advancements Comput. Technol.*, vol. 4, no. 11, pp. 326–335, 2012.
[34] W. T. Pan, "Estimate the applications of modified fruit fly optimization algorithm feasibility," in *Proc. 2nd Int. Conf. Ind. Design Mech. Power Appl. Mech. Mater. (ICIDMP)*, vol. 437, 2013, pp. 845–848.
[35] D. Niu, H. Wang, H. Chen, and Y. Liang, "The general regression neural network based on the fruit fly optimization algorithm and the data inconsistency rate for transmission line icing prediction," *Energies*, vol. 10, no. 12, p. 2066, 2017.
[36] J. Xu, Z. Wang, C. Tan, L. Si, and X. Liu, "A novel denoising method for an acoustic-based system through empirical mode decomposition and an improved fruit fly optimization algorithm," *Appl. Sci.*, vol. 7, no. 3, p. 215, 2017.
[37] J. Xu, Z. B. Wang, C. Tan, L. Si, L. Zhang, and X. H. Liu, "Adaptive wavelet threshold denoising method for machinery sound based on improved fruit fly optimization algorithm," *Appl. Sci.*, vol. 6, no. 7, p. 199, Jul. 2016.
[38] B. Ramachandran and G. T. Bellarmine, "Improving observability using optimal placement of phasor measurement units," *Int. J. Elect. Power Energy Syst.*, vol. 56, pp. 55–63, Mar. 2014.
[39] L. Wang, X. L. Zheng, and S. Y. Wang, "A novel binary fruit fly optimization algorithm for solving the multidimensional knapsack problem," *Knowl.-Based Syst.*, vol. 48, no. 2, pp. 17–23, 2013.
[40] T. Meng and Q.-K. Pan, "An improved fruit fly optimization algorithm for solving the multidimensional knapsack problem," *Appl. Soft Comput.*, vol. 50, pp. 79–93, Jan. 2017.
[41] L. Wang, R. Liu, and S. Liu, "An effective and efficient fruit fly optimization algorithm with level probability policy and its applications," *Knowl.-Based Syst.*, vol. 97, pp. 158–174, Apr. 2016.
[42] X. Han, Q. Liu, H. Wang, and L. Wang, "Novel fruit fly optimization algorithm with trend search and co-evolution," *Knowl.-Based Syst.*, vol. 141, pp. 1–17, Feb. 2018.
[43] T. Li, L. Gao, P. Li, and Q. Pan, "An ensemble fruit fly optimization algorithm for solving range image registration to improve quality inspection of free-form surface parts," *Inf. Sci.*, vols. 367–368, pp. 953–974, Nov. 2016.
[44] N. Mhudtongon, C. Phongcharoenpanich, and S. Kawdungta, "Modified fruit fly optimization algorithm for analysis of large antenna array," *Int. J. Antennas Propag.*, vol. 2015, Jun. 2015, Art. no. 124675.
[45] A. Darvish and A. Ebrahimzadeh, "Improved fruit-fly optimization algorithm and its applications in antenna arrays synthesis," *IEEE Trans. Antennas Propag.*, vol. 66, no. 4, pp. 1756–1766, Apr. 2018.
[46] J. Xu, Z. Wang, J. Wang, C. Tan, L. Zhang, and X. Liu, "Acoustic-based cutting pattern recognition for shearer through fuzzy C-means and a hybrid optimization algorithm," *Appl. Sci.*, vol. 6, no. 10, p. 294, 2016.
[47] Z. Qu, K. Zhang, J. Wang, W. Zhang, and W. Leng, "A hybrid model based on ensemble empirical mode decomposition and fruit fly optimization algorithm for wind speed forecasting," *Adv. Meteorol.*, vol. 2016, Aug. 2016, Art. no. 3768242.
[48] W.-T. Pan, "Using modified fruit fly optimisation algorithm to perform the function test and case studies," *Connection Sci.*, vol. 25, nos. 2–3, pp. 151–160, Jun. 2013.
[49] D. Shan, G. Cao, and H. Dong, "LGMS-FOA: An improved fruit fly optimization algorithm for solving optimization problems," *Math. Problems Eng.*, vol. 2013, Aug. 2013, Art. no. 108768.
[50] Q.-K. Pan, H.-Y. Sang, J.-H. Duan, and L. Gao, "An improved fruit fly optimization algorithm for continuous function optimization problems," *Knowl.-Based Syst.*, vol. 62, pp. 69–83, May 2014.
[51] X. Yuan, X. Dai, J. Zhao, and Q. He, "On a novel multi-swarm fruit fly optimization algorithm and its application," *Appl. Math. Comput.*, vol. 233, pp. 260–271, May 2014.
[52] Y. Zhang, G. Cui, J. Wu, W.-T. Pan, and Q. He, "A novel multi-scale cooperative mutation fruit fly optimization algorithm," *Knowl.-Based Syst.*, vol. 114, pp. 24–35, Dec. 2016.
[53] Y. Cong, J. Wang, and X. Li, "Traffic flow forecasting by a least squares support vector machine with a fruit fly optimization algorithm," *Procedia Eng.*, vol. 137, pp. 59–68, Dec. 2016.
[54] X.-L. Zheng and L. Wang, "A two-stage adaptive fruit fly optimization algorithm for unrelated parallel machine scheduling problem with additional resource constraints," *Expert Syst. Appl.*, vol. 65, pp. 28–39, Dec. 2016.
[55] S.-X. Lv, Y.-R. Zeng, and L. Wang, "An effective fruit fly optimization algorithm with hybrid information exchange and its applications," *Int. J. Mach. Learn. Cybern.*, vol. 9, no. 10, pp. 1623–1648, Oct. 2018.
[56] C. Xiao, K. Hao, and Y. Ding, "An improved fruit fly optimization algorithm inspired from cell communication mechanism," *Math. Problems Eng.*, vol. 2015, Dec. 2015, Art. no. 492195.
[57] L. Wang, Y. Xiong, S. Li, and Y.-R. Zeng, "New fruit fly optimization algorithm with joint search strategies for function optimization problems," *Knowl.-Based Syst.*, vol. 176, pp. 77–96, Jul. 2019.
[58] R. V. Rao, "Jaya: A simple and new optimization algorithm for solving constrained and unconstrained optimization problems," *Int. J. Ind. Eng. Comput.*, vol. 7, no. 1, pp. 19–34, 2016.
[59] R. V. Rao, K. C. More, J. Taler, and P. Ocłoń, "Dimensional optimization of a micro-channel heat sink using Jaya algorithm," *Appl. Therm. Eng.*, vol. 103, pp. 572–582, Jun. 2016.
[60] I. Babaoglu, "Artificial bee colony algorithm with distribution-based update rule," *Appl. Soft Comput.*, vol. 34, pp. 851–861, Sep. 2015.
[61] H. Haklı and H. Uğuz, "A novel particle swarm optimization algorithm with Levy flight," *Appl. Soft Comput.*, vol. 23, pp. 333–345, Oct. 2014.
[62] A. Babalık, H. İşcan, İ. Babaoğlu, and M. Gündüz, "An improvement in fruit fly optimization algorithm by using sign parameters," *Soft Comput.*, vol. 22, no. 22, pp. 7587–7603, 2018.
[63] M. R. Bonyadi and Z. Michalewicz, "SPSO 2011: Analysis of stability; Local convergence; And rotation sensitivity," in *Proc. Annu. Conf. Genetic Evol. Comput.*, 2014, pp. 9–16.

HAZIM ISCAN was born in Turkey, in April 1973. He graduated from the Department of Computer Systems, Gazi University, Turkey, in 1996. He received the M.S. degree from the Computer Engineering Department, Selcuk University, in 2004, where he is currently pursuing the Ph.D. degree. He has been a Research Staff at the Department of Computer Engineering, Konya Technical University, since 1997. His research interests include optimization, nature-inspired algorithms, machine learning, and artificial intelligence systems.

MUSTAFA SERVET KIRAN received the B.S. and Ph.D. degrees in computer engineering from the Institute of Natural and Applied Sciences, Selcuk University, Konya, Turkey, in 2010 and 2014, respectively. He is currently an Associate Professor with the Computer Engineering Department, Konya Technical University. His current research interests include swarm intelligence, evolutionary algorithms, and their real-world applications.

MESUT GUNDUZ received the B.S. and Ph.D. degrees in computer engineering from the Institute of Natural and Applied Sciences, Selcuk University, Konya, Turkey, in 2001 and 2006, respectively. He is currently an Associate Professor with the Computer Engineering Department, Konya Technical University. His current research interests include routing in computer networks, swarm intelligence, evolutionary algorithms, and their applications in networks.