



Grey Wolf Optimizer Based Tuning of a Hybrid LQR-PID Controller for Foot Trajectory Control of a Quadruped Robot

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Abstract

Quadruped robots have generally complex construction, so designing a stable controller for them is a major struggle task. This paper presents designing and optimization of an effective hybrid control by combining LQR and PID controllers. In this study, the tuning of a hybrid LQR-PID controller for foot trajectory control of a quadruped robot during step motion using Grey Wolf Optimizer (GWO) algorithm which is an alternative method are comparatively investigated with two traditional benchmarking algorithms (PSO and GA). The principal goal of this work is the tuning of the LQR controller parameters (Q and R weight matrices) and the PID controllers gains (k_p , k_i and k_d) using the proposed algorithms. Initially, the designed solid model of the quadruped robot is imported into Simulink/SimMechanics which are simulation tools of MATLAB and then obtained the mathematical model of system which is at State-Space form with Linear Analysis Tools considering the step motion of robot leg in sagittal plane. Later, the hybrid LQR-PID control system is designed and its parameters are tuned to get optimal values which guarantee best trajectory tracing in Simulink with the three proposed algorithms. Subsequently, the system is simulated separately with optimal control parameters which provide from the algorithms. The simulation outcomes are indicating that GWO algorithm is more efficiently and quickly within similar torques to tuning the hybrid controller based on LQR&PID than the other conventional algorithms.

1. INTRODUCTION

Quadruped robots fall under the classification of legged robots and they are a significant area in robotic with the increasing popularity in the last decade. The advantages of quadruped robots such as good stability, high flexibility, and locomotion on uneven terrain make it more attractive than tracked and wheeled robots. There are many accomplished quadruped robots developed in recent years. Some of the foremost ones are following. BigDog [1] is a basic milestone which developed by Boston Dynamics. ANYmal [2] and StarLETH [3] are developed by Hutter et al. in ETHzurich, they are driven by electric actuators. Jinpoong [4] is developed by Cho et al. in KITECH, it is driven by hydraulic actuators. Another robot driven by hydraulic actuators, HyQ2Max [5] is developed by Semini et al. in IIT.

The complexity of the system makes it difficult to control and requires the development of different controllers. Traditional PID controller is extensively used in many engineering applications because of the important effectiveness and the easy to implement. Hutter et al. [6] suggested a cascade controller which include positions and torque controllers using PID controller and feedback friction compensation for ANYmal. Chang et al. [7] proposed a Fuzzy-PID control algorithm for a quadruped robot driven by hydraulic actuators to tuning of gains in real-time condition as comparatively with traditional PID controller. They denoted that the proposed algorithm which ensure the joint positions control of robot is more robustness and more adaptive. LQR controller is also ensure a very famous and handlely control algorithm for working-out of difficulties. Hubacher [8], to overcome with unstable system dynamics, proposed an LQR based adaptive control framework which can updating the low-level controller by guessing the current system dynamics. Meng et al. [9] present a balancing method control to stabilize of a quadruped robot in motion using LQR controller. Focchi et al. [10] designed both PID and LQR controllers

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for a hydraulically-driven quadruped robot leg, respectively. They denoted that the performance of LQR controller decreases at higher frequencies.

In designing of PID controller, gains are determined commonly with trial-error methods and Ziegler-Nichols method. Similarly, in the design of the LQR controller, state (Q) and control (R) weighting matrices are selected by designer using traditional methods. However, defining PID controller gains and LQR controller weighting matrices are a significant duty and should be adjust with a search tools (in particular with tuning algorithms). There are many studies in literature about the optimization of PID and LQR controllers based on intelligent optimization and soft computing technique. For example, GWO-Grey Wolf Optimizer algorithm [11,12], PSO-Particle Swarm Optimization [13,14] and GA-Genetic Algorithm [15,16] are used for optimization of PID and LQR controllers. Also, in previous study [17] by authors, adjusting of PID controller using GWO, PSO and GA for foot trajectory control during step motion of quadruped robot is investigated in detail.

The hybrid controller based on LQR&PID is pre-designed for foot trajectory control of a quadruped robot during step motion and the control parameters are tuned with GWO, PSO and GA in this paper. The controller parameters consist of the weight matrices (Q and R) of LQR controller and the gains of PID controllers. Firstly, the solid model of the quadruped robot is built at CAD software SolidWorks and imported into Simulink/SimMechanics which are simulation tools of MATLAB. Linear Analysis Tool (LAT) is used for obtaining the linear model (mathematical) of system. Finally, the controller parameters are adjusted offline in MATLAB/Simulink and the system is run separately with optimal control parameters which provide from the algorithms. Eventually, to show the achievement of recommended search methods, the simulation outcomes are presented in graphical forms.

Following from overview in the introduction, the sections of paper is constituted as follows respectively: the linear modelling of the system is explained, the proposed algorithms are described briefly, the designing of the optimal hybrid LQR-PID controller is present, the simulation results are shown comparatively in graphical forms and the paper finalized at conclusions.

2. OBTAINING LINEAR MODEL OF THE SYSTEM

This part of paper presents the physical specifications of the quadruped robot and the linear modelling method of system. Modelling as mathematical of a quadruped robot is difficult task due to its complex structure. Therefore, the quadruped (four-legged) robot is similarly modelled with other legged robots using SolidWorks software and imported into SimMechanics. The structure of robot is simple and allows performing a wide range of tasks. The robot consists of a main torso and four legs with 3-DoF which are mounted the torso. Each leg has three links which are actuated by rotating joints.

The solid model of quadruped robot with the physical parameters is given, in Figure 1 and in Table 1, respectively. The physical parameters are defined as the inspiration from walking animals in nature and they are similar with other quadruped robots in literature.

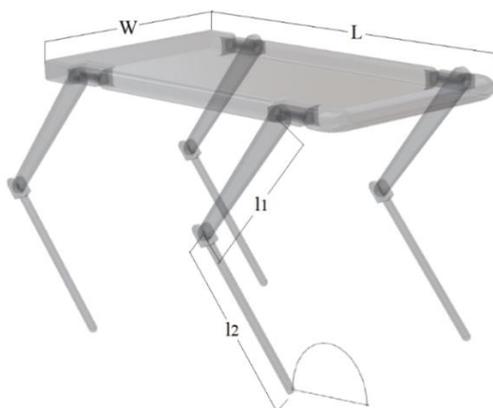


Figure 1. Solid model of the quadruped robot

Table 1. The main parameters of the robot

W (width of Torso)	500 mm
L (length of Torso)	1000 mm
l1 (length of Hip link)	400 mm
l2 (length of Knee link)	410 mm
hs (height of a single step)	100 mm
ls (size of a single step)	200 mm
Total Weight of System and Material	68.3 kg Alloy 1060

In scope of this study, the foot trajectory control of tip point for one leg in the planner motion (sagittal plane) is considered by neglecting the degree of freedom on arm to provide the rotational motion of body. Since the all legs have the same dynamic characteristics, the control of trajectory of a one leg of the quadruped robot during step motion is considered in this study. The designed controller for a one leg can be also applied to the other legs. Thus, it is considered as a 2-DoF system which is consist of Hip (upper link) and Knee (lower link) joints.

The MATLAB/SimMechanics model of robot is adequate to design PID controller but a linear model is needed to achieve the LQR controller. For this reason, obtained the linear model of system via the SimMechanics model using LAT. After linearization, the mathematical equations in matrix form of State-Space model are obtained as follows;

$$\dot{X} = Ax + Bu, y = Cx + Du \quad (1)$$

$$x^T = [\theta_{Hip} \dot{\theta}_{Hip} \theta_{Knee} \dot{\theta}_{Knee}], y^T = [\theta_{Hip} \theta_{Knee}], u^T = [\tau_{Hip} \tau_{Knee}] \quad (2)$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -2,63e^{-12} & 0 & 1,30e^{-16} & 0 \\ 0 & 0 & 0 & 1 \\ -3,50e^{-12} & 0 & 2,45e^{-15} & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 10,41 & 11,90 \\ 0 & 0 \\ 11,90 & 66,11 \end{bmatrix}, C = \begin{bmatrix} 57,29 & 0 & 0 & 0 \\ 0 & 57,29 & 0 & 0 \\ 0 & 0 & 57,29 & 0 \\ 0 & 0 & 0 & 57,29 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (3)$$

The flowchart of this study which include modelling of system, pre-design and optimization of the hybrid LQR-PID controller is given in Figure 2.

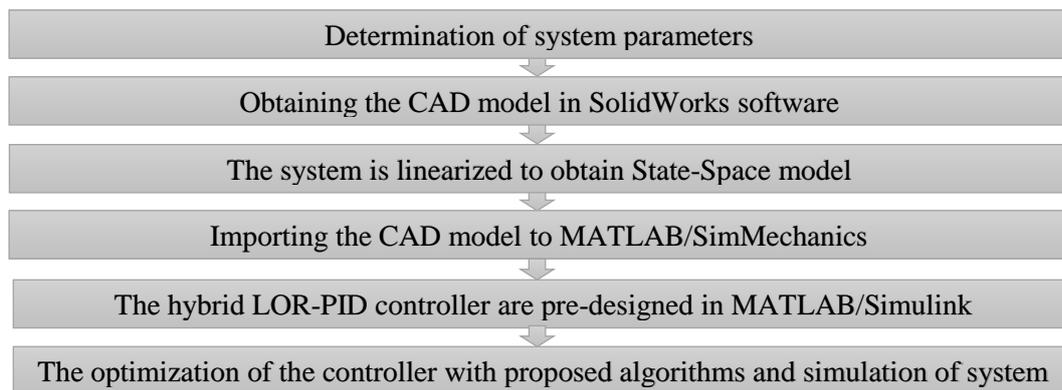


Figure 2. The flowchart of modelling, optimization and simulation of system

3. DESCRIPTION OF THE RECOMMENDED ALGORITHMS

Introductions of GWO, PSO and GA methods are briefly summarized in this section. The proposed algorithms are used to adjusting of the all controller parameters of the hybrid LQR-PID controller which is ensure trajectory control of the leg. PSO and GA are used as benchmarking tools to evaluate of GWO.

3.1. Description of Grey Wolf Optimizer

GWO [18] algorithm is a search and tuning method which is simulates the hunting mechanism of grey wolves within the hierarchy of leadership, it is proposed by Mirjalili et al. in 2014. The search optimization process is based on mimicking behavior of grey wolves during hunting and sorting solutions by hierarchy in group. There are many studies [19-26] are available in which the GWO algorithm is investigated in detail.

The technique simulates the social hierarchy and hunting behavior in the society of grey wolves in the wild. In grey wolf hierarchy, grey wolves are examined in 4 different simulation groups according to their dominance status between each other. As you can see in Figure 3, the simulations are Alpha (α) which is as leader, Beta (β), Delta (δ) and Omega (ω) as least weak, hierarchical respectively. The Alpha (α) wolf is the leader which responsible for making decisions about hunting and other activates (sleeping time and

areas in territories etc.) in grey wolves' group, so it is highest ranking grey wolf in the hierarchy. As an assistant of Alfa (α) wolf at authorized to make decision, Beta (β) wolf is the second rank in hierarchy. Following Alpha (α) and Beta (β) wolves, Delta (δ) is lowest ranking grey wolf but dominate Omega (ω). Other wolves except Alpha (α), Beta (β) or Delta (δ) wolves are called Omega (ω) in grey wolf hierarchy.

In optimization process using GWO, the quest starts with potential solutions (initial population of wolves) which are generated randomly. In during the hunting (adjusting of parameters), these wolves guess the location of prey (optimal) through an iterative procedure. Alpha (α) is the fittest, Beta (β) is the second-best solution and Delta (δ) is the third best solutions. Other solutions expert the solutions are least significant and they are considered as Omega (ω) lastly [18].

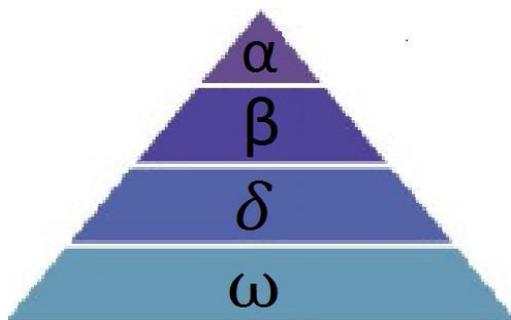


Figure 3. Grey wolf hierarchy [18]

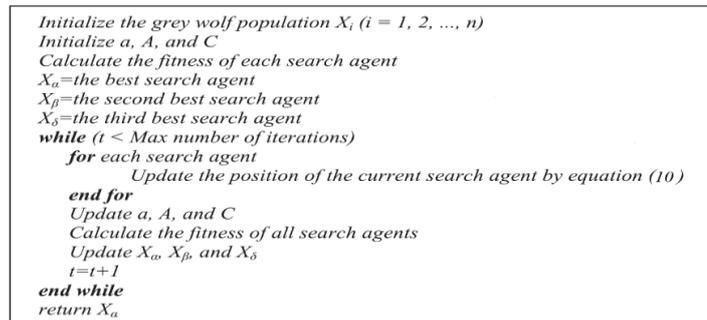


Figure 4. Flowchart of GWO algorithm [18]

In GWO algorithm, there are three steps in the hunting behavior of grey wolves mainly. The hunting process starts with tracking (follow up at some distance), chasing (pursuit for a short time) and approaching (nearing by encircling) of the prey. Harassing and encircling of the prey continues process until the prey stops remains still. At last, wolves are attacked the prey. The behavior of grey wolves in the hunting steps according to the social hierarchy are mathematically modelled and described by Mirjalili et al in detail [18]. The mathematically model introduced briefly following: The encircling behavior of grey wolves is mathematical modelled as,

$$\vec{D} = |\vec{C}\vec{X}_p(\text{iter}) - \vec{X}(\text{iter})| \quad (4)$$

$$\vec{X}(\text{iter} + 1) = \vec{X}_p(t) - \vec{A}\vec{D} \quad (5)$$

\vec{X}_p and \vec{X} are represents position vector of prey and grey wolf, respectively. iter is the current iteration. The \vec{A} and \vec{C} are coefficient vectors, they are calculated as,

$$\vec{A} = 2\vec{a}\vec{r}_1 - \vec{a} \quad (6)$$

$$\vec{C} = 2\vec{r}_2 \quad (7)$$

\vec{r}_1 and \vec{r}_2 are random vectors which ranges is in the [0, 1]. Components of \vec{a} are linearly decreased from 2 to 0 through the iterations. The hunting of grey wolves is mathematically modelled as,

$$\vec{D}_\alpha = |\vec{C}_1\vec{X}_\alpha - \vec{X}| \quad \vec{D}_\beta = |\vec{C}_2\vec{X}_\beta - \vec{X}| \quad \vec{D}_\delta = |\vec{C}_3\vec{X}_\delta - \vec{X}| \quad (8)$$

$$\vec{X}_1 = \vec{X}_\alpha - \vec{A}_1(\vec{D}_\alpha) \quad \vec{X}_2 = \vec{X}_\beta - \vec{A}_2(\vec{D}_\beta) \quad \vec{X}_3 = \vec{X}_\delta - \vec{A}_3(\vec{D}_\delta) \quad (9)$$

$$\vec{X}(\text{iter} + 1) = (\vec{X}_1 + \vec{X}_2 + \vec{X}_3)/3 \quad (10)$$

Initially, there is no prior knowledge about the optimal parameters and their place in search area. The first obtained 3 best solutions (α β δ) are assumed that they have better knowledge about location of prey.

Throughout iterations, α β and δ guess the possible position of the victim and updates its distance. Then, the other agents are forced that updating their positions according to the best solutions. Potential solutions tend to leave from prey (If $|\vec{A}| > 1$) or converge towards the prey (If $|\vec{A}| < 1$). Lastly, the GWO is finished by an end criterion or iterations numbers. The flowchart of GWO algorithm is given in Figure 4.

3.2. Description of Particle Swarm Optimization

PSO [27] is one of the most popular optimization algorithms which is inspired by the social behavior of birds or fish. In 1995, it is improved by Eberhart and Kennedy. In recent years, the PSO algorithm has been applied many in practice such as mathematical problems, scientific optimization problems and very special engineering areas [28-30].

The traditional PSO algorithm starts with a swarm (first population at start) of particle (potential solution). Particles search across the search area via defined formulations. After research in the search field, particles move to their best-known positions. The particles will guide the movement of other particles after finding the best position. The search for the search field is repeated until the satisfactory solution is finally found. In each iteration, the swarm is tuned according to the equations in the following;

$$v_i^{t+1} = \omega v_i^t + c_1 r_1 (p_i^t - x_i^t) + c_2 r_2 (g_i^t - x_i^t) \quad (11)$$

$$x_i^{t+1} = x_i^t + v_i^{t+1} \quad i=1,2,\dots,n \quad (12)$$

t is the number of iteration, n is the particles number, C_1 and C_2 are the positive constants, w is the weighted inertia, r_1 and r_2 are two random numbers which are changing in the range $[0,1]$, p_i is the best position of particle and g_i is the best particle.

3.3. Description of Genetic Algorithm

GA [31] is a traditionally and extensively used tuning method based on genetics and natural selection. Moreover, it is preferred mostly as a benchmarking tool for investigated of novel algorithms. For GA optimization in this study, Global Optimization Toolbox in MATLAB [32] is used. It is a powerful tool to get solutions efficiently and effectively.

4. PRE-DESIGNING AND OPTIMIZATION OF THE HYBRID LQR-PID CONTROLLER

The description of the pre-designing and optimization of the hybrid LQR-PID controller is given in this section. PID [33] and LQR [34] controllers are two separate commonly used controller in control theory. PID controller involves three constant gains (k_p , k_i , k_d). The behavior of LQR controller is determined by two weighting matrices: state (Q) and control (R) matrices which are includes parameters that need to be adjust. The optimal matrices determine the gains matrix (K) of controller. The defining these parameters is an optimization problem and should be solving using optimization algorithms instead of trial-error methods.

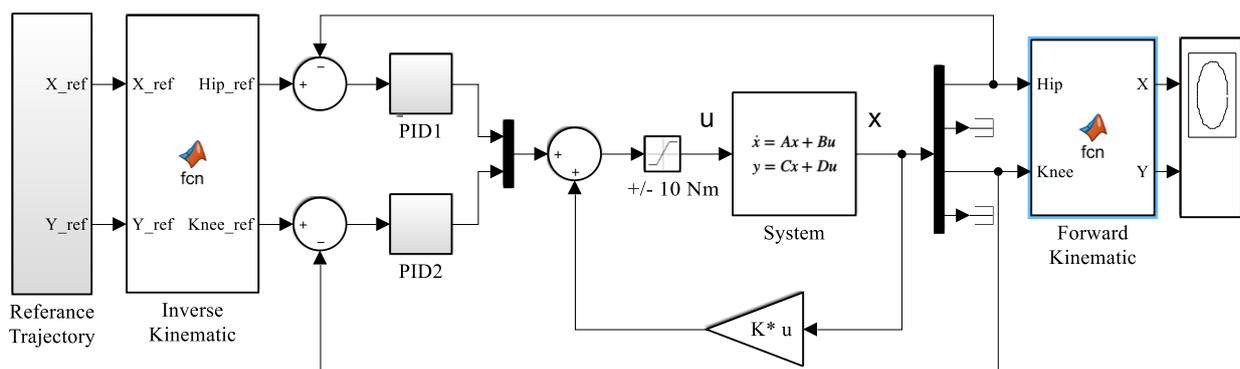


Figure 5. The hybrid LQR-PID controller model of system

In this study, a hybrid LQR-PID controller consisting of a combination of two PID controllers and LQR controllers is designed to get a better controller by taking advantage of the both controllers. The hybrid LQR-PID controller is pre-designed in MATLAB/Simulink and the control system is given Figure 5.

We introduced the inverse/forward kinematics of a quadruped robot in previous study [35]. The kinematics solutions are used to describe the mathematical relations between the angular positions of Hip/Knee joints and the trajectory coordinates of the leg tip point. That is, the foot trajectory control is achieved by controlling the joints. The reference trajectory (*hs*-step height, *ls*-step size) which is input of system is defined as a semicircle considering the physical constraints of robot [36,37].

The main goal is to obtain optimal control parameters of the hybrid LQR-PID controller which must be ensure foot trajectory tracking control of system. Therefore, determined a cost function which is including trajectory coordinates values. And, searched the control parameters with three different algorithms to minimize the cost function. The cost function (*J*) is:

$$J = \text{mean} \sqrt{(|x_{ref} - x|^2 + |y_{ref} - y|^2)} \quad (13)$$

x_{ref} and y_{ref} are the coordinates of the reference foot trajectory, x and y are the realized trajectory in single step movement at simulations. GWO, PSO and GA are employed for adjusting of the control parameters with respect to the proposed cost function. The tuning ranges of the parameters are set as denoted in Table 2. In Table 3, the parameters of the proposed algorithms are given.

Table 2. The tuning range of the parameters of hybrid LQR-PID controller

	LQR						PID 1			PID 2		
	Q ₁	Q ₂	Q ₃	Q ₄	R ₁	R ₂	k _p	k _i	k _d	k _p	k _i	k _d
Min	0	0	0	0	0.001	0.001	-2	-35	-850	-75	-100	-150
Max	70	5	1	100	10	15	2	35	850	75	100	150

Table 3. The parameters of proposed algorithms

GWO	Maximum Iteration	50	
	Number of Search Agent	20	
PSO	Number of Particle (NoP)	20	
	Max. Iteration	50	
	Inertia Weight	W _{init}	0.2
		W _{final}	0.9
	Velocity Clamping Parameter (V _{max})	6	
	Personal Best Value (c ₁)	2	
Neighborhood Best Value (c ₂)	2		
GA	Population Size	20	
	Elite Count	10	
	Generations	50	
	Crossover Fraction	0.8	

5. SIMULATION RESULTS

The proposed algorithms are applied to the hybrid LQR-PID controller in this section. Firstly, GWO and PSO are exported to MATLAB by coding. GA is already available in MATLAB Optimization Toolbox. The proposed algorithms are run on separately a PC with 16.0 GB memory along 50 iterations to get the minimum value of the cost function (*J*). After tuning, the best values of LQR and PID parameters set corresponding to the minimum cost function value provided by the optimal GWO, PSO and GA are given in Table 4. Moreover in Table 4, are shown minimum the cost function value (J_{\min}) and the elapsed time in optimizations.

The system simulated with tuned parameters of the hybrid LQR-PID control in MATLAB/Simulink. The simulation time is considered 2 seconds. The effect of the proposed algorithms on trajectory tracing performances comparatively showed in Figure 6. In this Figure, the hybrid LQR-PID controllers which tuned with GWO, PSO and GA ensure settling on the path very successfully with minimal errors. Even if the trajectory tracing performances of the proposed algorithms are similarly, they traced trajectory with different errors value. As seen clearly in Figure 7 and in Figure 8, GWO performed trajectory with less errors than PSO and GA in spite of similar hip (h) and knee (k) joints torque input values. Furthermore, the convergence performances of the GWO, PSO and GA are shown in Figure 9. It is clearly obvious from this figure that GWO is better both at the initial and at the end. PSO and GA are converged similar to each other but slower than the GWO. As a result, the GWO is providing better response than the PSO and GA by reducing the cost function more.

Table 4. The tuned parameters of the hybrid LQR-PID controller

		GWO	PSO	GA
LQR	Q_1	12.22	47.06	13.21
	Q_2	0.99	11.27	2.89
	Q_3	0.78	0.0006	0.17
	Q_4	45.74	22.83	4.63
	R_1	6.88	7.56	6.65
	R_2	14.56	23.05	6.85
	K	[1.30 0.67 0.06 0.50; -0.18 -0.10 0.22 1.73]	[2.47 1.43 0.001 0.22; -0.18 -0.12 0.005 0.98]	[1.40 0.87 -0.001 -0.01; 0.009 -0.02 0.15 0.82]
PID1	k_p	-1.51	5.15	1.48
	k_i	5.24	-15.72	3.99
	k_d	680.11	566.44	733.03
PID2	k_p	-66.90	-2.35	-12.53
	k_i	72.75	9.28	-10.13
	k_d	110.28	67.80	144.07
J_{min}		0.4624	1.2100	1.4563
Elapsed time (s)		1965.4	1314.5	1897.2

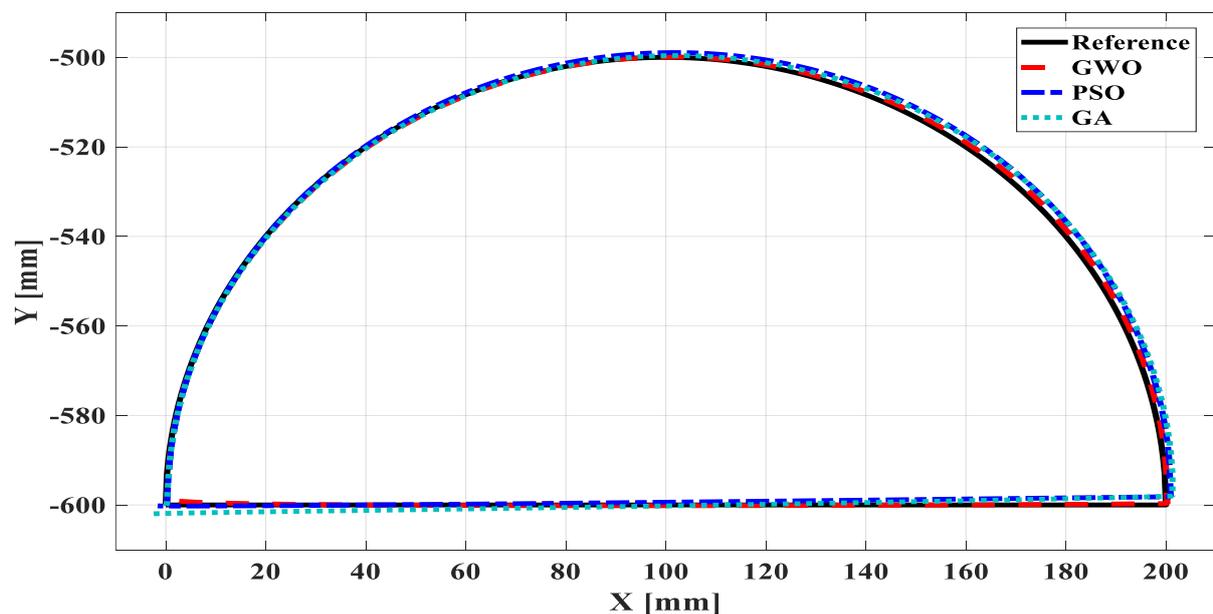


Figure 6. The trajectory control performance of the tuned LQR-PID controller

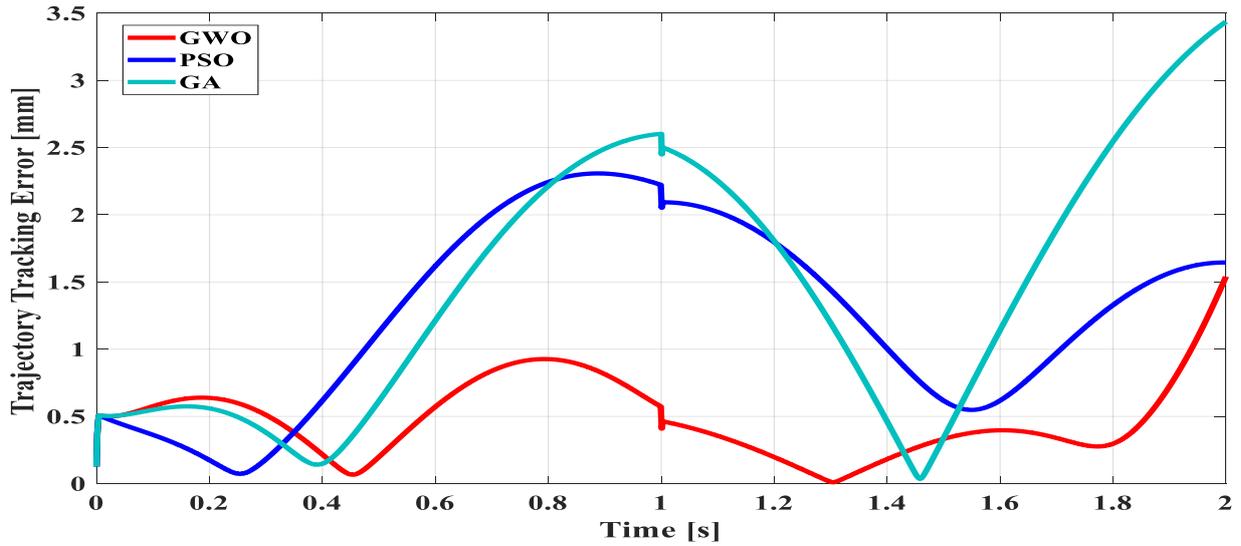


Figure 7. The absolute trajectory errors of the proposed algorithms in xy plane

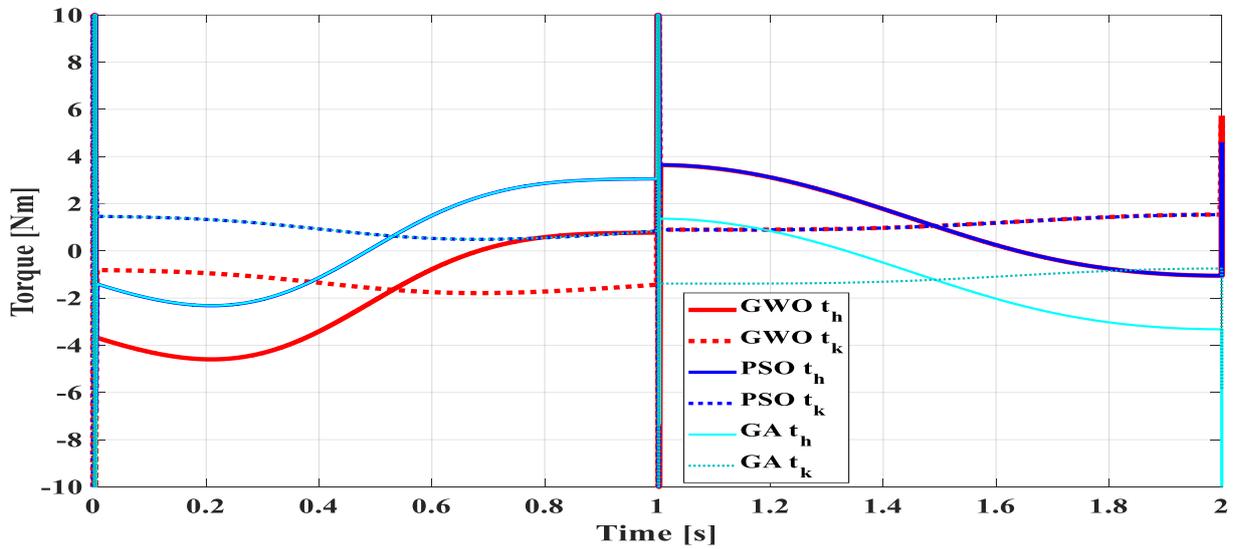


Figure 8. The input torques for hip(h) and knee(k) joints of the proposed algorithms

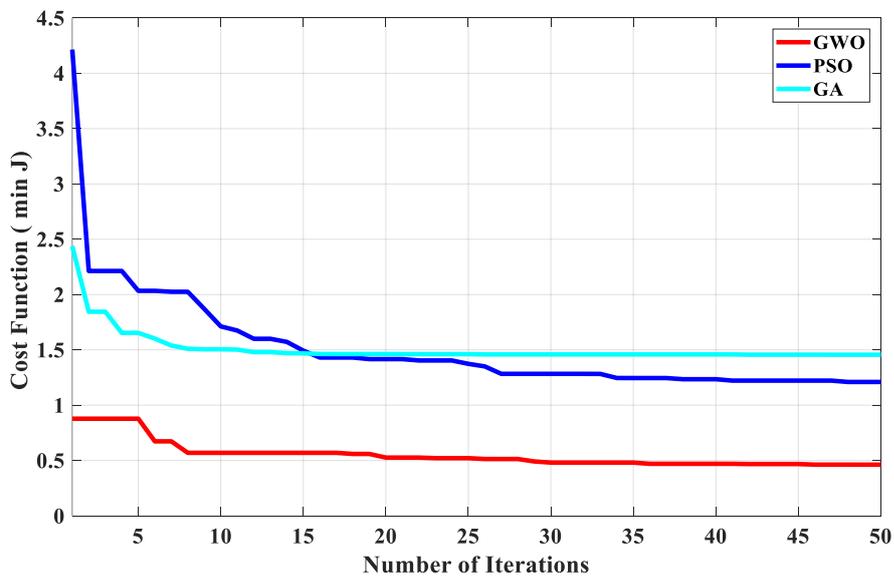


Figure 9. The performances of proposed algorithms in reducing cost function

6. CONCLUSIONS

In this paper, the tuning of a hybrid LQR-PID controller for foot trajectory control during step motion of a quadruped robot using GWO which is an alternative method are investigated in comparative with two widespread search algorithms which are PSO and GA. The main aim of the study, unlike studies in the literature, development a diverse optimal controller which is consist of combination LQR and PID controller for quadruped robots by optimizing it with GWO which is a diverse search algorithm. Moreover, to investigate the effectiveness of GWO on the control system, it is compared with PSO and GA. The proposed algorithms are successfully operated to adjusting the LQR controller's weighting matrices and the PID controller's gains. The simulation results show that the tuned hybrid LQR-PID controller with all proposed methods separately can perform tracing trajectory with different errors values. However, the trajectory tracking errors and performances analyses of the proposed algorithms show that GWO is the best method in this study because of it is ensure more satisfactory performances from PSO and GA.

CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

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